Dynamic Seller Competition When Buyers Have Bargaining Power

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Abstract

Existing models of dynamic competition assume that buyers are price-takers, despite price negotiation being common in some of the industries used to motivate this literature. We extend a well-known duopoly model of competition where learning-by-doing lowers sellers' costs to allow for Nash-in-Nash bargaining. We analyze how different levels of buyer bargaining power and dynamics combine to change market structure, efficiency and the recommendations of two stylized policy analyses. We identify how moderate buyer bargaining power can increase the dynamic incentives of a leader relative to those of a laggard, as the leader will expect its lead to last longer. This can cause predicted outcomes and policy recommendations to be quite different from predictions that ignore dynamics or ignore bargaining power.

Keywords: dynamic competition, learning-by-doing, bargaining power, buyer power, multiple equilibria, dynamic incentives, subsidies, horizontal mergers.

JEL codes: C73, D21, D43, L13, L41.

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1 Introduction

Industries where learning-by-doing (LBD), switching costs or network effects allow firms that make current sales to gain future competitive advantages are often the focus of government policies. Some of these policies, such as subsidy or trade policies, aim to speed up the accumulation of competitive advantages, while others, including antitrust policies against predatory pricing or mergers, may limit how far a leader gets ahead. Analysis of these industries requires economic models that are both tractable and capture those real-world features that determine whether advantage accumulation is too fast or too slow.

In this paper we examine how allowing buyers to have bargaining power over prices affects predicted model outcomes and stylized policy counterfactuals. We have two primary motivations. First, anecdotes suggest that negotiation over prices is widespread in several capital or intermediate goods industries where economists have found LBD and dynamic competition to be important, even though almost all of the existing dynamic competition literature assumes buyers are price-takers.[1] Second, while recent studies using static models of competition often allow for bargaining, there are reasons to think that the effects of negotiations could be different in a dynamic model. For example, when suppliers' costs are fixed, an increase in buyer bargaining power will tend to shrink a lower cost seller's markup more than those of its rivals, and so tend to increase efficiency. However, with LBD, efficiency also depends on sellers' incentives to invest to lower their future costs (Besanko, Doraszelski, and Kryukov (2019)). Given that learning when sellers set prices may already be too slow (Cabral and Riordan (1994)), a reduction in sellers' profits could, plausibly, further reduce their incentives to invest, leading to further efficiency loss.

Our analysis extends the "learning-and-forgetting" dynamic competition model of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS hereafter) to allow for bargaining. In

[1] For example, press reports suggest that buyers, especially larger buyers, can negotiate lower prices for commercial aircraft (`https://simpleflying.com/how-much-do-boeing-aircraft-cost/`), studied by Benkard (2004), Kim (2014) and An and Zhao (2019); wind turbines (where turbine services agreements are individually negotiated, `https://www.stoel.com/insights/reports/the-law-of-wind/design-engineering-construction-and-procurement`), studied by Covert and Sweeney (2022); and, semiconductors (`https://www.linkedin.com/pulse/latest-news-semiconductor-foundries-negotiating-price-quincy-qiu/`), studied by Siebert (2010).

the BDKS model there are two differentiated sellers. Each seller has M possible "know-how" states. Know-how accumulates through sales to atomistic single-unit buyers arriving each period, but it can also stochastically depreciate. Assuming sellers set prices, BDKS show how numerical homotopy methods can be used to enumerate a set of Markov Perfect Nash equilibria (MPNE). They also show that the combination of LBD and depreciation can lead to different outcomes than simply slower learning, and that, when depreciation is possible, multiple equilibria with distinct competitive dynamics are common.

We allow for each seller's price to be determined by Nash-in-Nash bargaining (Horn and Wolinsky (1988), Collard-Wexler, Gowrisankaran, and Lee (2019)), with the buyer observing its idiosyncratic preferences and choosing which seller to buy from after prices have been negotiated.[2] Assuming the buyer bargaining weight parameter, τ, equals zero is equivalent to BDKS's seller price-setting assumption, while $\tau = 1$ leads to a unique equilibrium where all prices equal production costs. The Nash-in-Nash formulation has been widely adopted in static models (e.g., Grennan (2013), Gowrisankaran, Nevo, and Town (2015), Ho and Lee (2017) and Crawford, Lee, Whinston, and Yurukoglu (2018)), but our context has the additional feature that the bargaining weight affects sellers' outside options because of how it changes future negotiations and market structure. This creates the possibility of feedback loops between bargaining, market structure and dynamic incentives which models without dynamics or without bargaining would miss.

We focus on two model specifications. We derive several novel comparative statics results analytically assuming $M = 2$, i.e., sellers can have either high or low costs. Fudenberg and Tirole (1983) and Leahy and Neary (1999) also use two-period special case models to derive results, although they do not allow for know-how depreciation. We use BDKS's $M = 30$ specification, solved for different bargaining weights, to examine how these comparative statics generalize and to analyze subsidy policies.[3] Our analysis of the $M = 30$ model is facilitated by our (numerical) finding

[2] Dorn (2025) uses Kalai (1977) proportional bargaining to model multiperiod hospital-insurer contracts with index clauses. Considering alternatives to Nash-in-Nash bargaining would be an interesting extension to our analysis.

[3] Our analysis of a horizontal merger will use an $M = 7$ model with 4 products. Our working paper, Deng, Jia, Leccese, and Sweeting (2025), contains some additional results for a specification with $M = 3$.

that MPNEs are unique if $\tau \geq 0.25$, so that a feature of MPNE outcomes that can make analysis difficult when sellers set prices is eliminated when buyers have more than modest bargaining power.

The first step in our analysis considers market concentration and dynamic incentives. Concentration will tend to increase when know-how leaders are more likely to make sales in asymmetric states, reflecting changes in the difference between the leader's and the laggard's equilibrium prices. We decompose these price gaps into differences in production costs, differences in static markups and differences in dynamic incentives. If the leader is more likely to win when $\tau = 0$, then, holding dynamic incentives fixed, an increase in τ will change static markups in a way that makes the leader even more likely to win. Therefore, how dynamic incentives change can play a key role in how bargaining power affects market structure.

We show that, in the $M = 2$ model, the leader's dynamic incentive will increase relative to the laggard's when τ is small so that this channel will also lead to concentration increasing. This occurs even though increases in τ shrink the leader's share of surplus in future bargains, as the intuition laid out above suggested, because anticipated changes in future negotiations will result in the leader expecting its lead to last longer if it wins the current sale. The static markup and dynamic incentive effects combine to cause concentration to increase monotonically in τ in the $M = 2$ model. We find that in the $M = 30$ specification the leader's dynamic incentives also increase relative to those of the laggard when τ is small as long as depreciation is not too likely. As a result, a model with dynamic sellers and moderate buyer bargaining power can predict much higher concentration than models where either buyers are price-takers or sellers' dynamic incentives are ignored.

The probability that know-how leaders make sales also affects economic distortions and efficiency. Following Besanko, Doraszelski, and Kryukov (2019), we decompose the difference between equilibrium and social planner surplus into losses from the failure to maximize surplus in the states visited given equilibrium play (a "current" distortion) and losses due to differences in how the industry evolves (a "transition" distortion). The sign of the current distortion typically changes as buyer bargaining power increases, and, if depreciation is not too likely, moderate levels of buyer bargaining power minimize both distortions in the $M = 30$ model, so that the equilibrium comes close to being efficient. In contrast, a model without seller dynamic incentives predicts that efficiency would be maximized if buyers had all, or almost all, of the bargaining power.

We assume parameters that imply significant LBD and limited know-how depreciation to analyze two stylized policy problems. The first problem is the design of a tax and subsidy scheme to maximize efficiency. We show that accounting for equilibrium dynamic incentives and buyer bargaining power can change the direction and the size of the optimal transfers in many states, and that, when we account for dynamics, a scheme that can be optimal with price-setting sellers is worse than no scheme when buyers have even small amounts of bargaining power. Our second problem is the evaluation of a hypothetical horizontal merger between know-how leaders in an expanded version of the model with four products, up to four firms and seven know-how states for each product. Unlike An and Zhao (2019), who focus on know-how efficiencies in the Boeing-McDonnell Douglas merger, we assume the merger generates no efficiencies or fixed cost savings. If dynamic incentives or buyer bargaining power are ignored, the proposed merger lowers efficiency and consumer surplus, whereas with both dynamics and some levels of moderate buyer bargaining power, the merger can raise efficiency or consumer surplus. Accounting for dynamic incentives and buyer bargaining power is also necessary to understand whether the merger will increase the expected variable profits of the parties. Both policy examples therefore illustrate how accounting for how dynamic incentives and bargaining power interact can be important.

Our conclusions come from a stylized model with restrictive functional form and economic assumptions. However, we believe that our model is an appropriate starting point for investigating how buyer bargaining power affects dynamic competition. We view the BDKS model, which allows for both know-how depreciation and LBD, as generalizing earlier stylized theoretical models with only LBD, such as Fudenberg and Tirole (1983) and Cabral and Riordan (1994), and capturing important features of models that have been applied to, among others, the commercial aircraft and space launch industries (Benkard (2004), An and Zhao (2019) and Su, Yang, and Sweeting (2026)).[4] A different model might consider the effects of bargaining where dynamic competition

[4] Benkard (2004) and An and Zhao (2019) assume sellers set quantities, whereas our model assumes they set prices,

is driven by demand-side network effects or switching costs (Farrell and Klemperer (2007), Cabral (2011)). However, industries with network effects and switching costs often, but not always, involve sales to final consumers at posted prices. Therefore, studying the effects of bargaining in a model where dynamics come from LBD seems an appropriate starting point.[5] Our model assumes sellers are unable to enter or exit. If we allowed for entry, exit and scrap values, the industry might disappear entirely once buyer bargaining power is sufficiently large, so we view our choice of model as allowing us to focus on how the full range of buyer bargaining power can affect competition between sellers. Finally, our model can be extended in various ways. For example, each buyer is assumed to be atomistic. Our working paper, Deng, Jia, Leccese, and Sweeting (2025), applying an approach developed in Sweeting, Jia, Hui, and Yao (2022) to allow for buyers to partially internalize their effects on future surplus, shows that several of our results extend when buyers are both forward-looking and have bargaining power.

After a discussion of the related literature, the rest of the paper is structured as follows. Section 2 explains how we extend the BDKS model to allow for bargaining. Section 3 characterizes equilibria and provides our results on when equilibria are unique. Section 4 shows how accounting for dynamic incentives and buyer bargaining power affects equilibrium market structure and efficiency. Section 5 considers our subsidy and merger counterfactuals.[6] Section 6 concludes. The online Appendices contain proofs of analytical results, computational details, and supporting numerical results.

Related Literature. The key feature of models of dynamic competition is that current sales affect a seller's future competitiveness. This distinguishes this literature from work on games where dynamics come from firm entry and exit, durable investment or mergers, but variable profits are determined by static price or quantity competition. Some papers in this latter literature, such as Yang (2020), have allowed for Nash-in-Nash bargaining in the static competition stage, either in

to allow for many aircraft to be sold in each period. Su, Yang, and Sweeting (2026) assume that providers of space launches compete in low-bid scoring procurement auctions with a long-lived government buyer, who has the option not to purchase a launch.

[5]Cloud computing may provide a B2B environment of growing importance where network effects and switching costs are present (Biglaiser, Crémer, and Mantovani (2024)).

[6]Deng, Jia, Leccese, and Sweeting (2025) also consider the effects of other policy tools that might be used to limit single firm dominance and the effects of exclusive contracts.

a baseline model or as a robustness check.

Market concentration, efficiency and the design of subsidy policies have been important topics in the existing dynamic competition literature. For example, Fudenberg and Tirole (1983) solve models where, in equilibrium, firms are identical, to examine how strategic incentives affect the level and time-paths of output, efficiency and the scope for tax and subsidy schemes to improve welfare.[7] Leahy and Neary (1999) also consider infant-industry subsidy schemes, with a focus on the role of government commitment (our government will be able to commit). Dasgupta and Stiglitz (1988) allow for one firm to have an initial advantage in order to understand whether such an industry will tend towards monopoly. Cabral and Riordan (1994), BDKS and Besanko, Doraszelski, and Kryukov (2014) also analyze whether a single firm is likely to become dominant, assuming, as we do, that there is a sequence of atomistic single-unit buyers so asymmetries arise naturally.

We use BDKS's model, which allows for know-how depreciation, and, when $M = 30$, we also use their computational tools to look for multiple equilibria. Besanko, Doraszelski, and Kryukov (2014) and Besanko, Doraszelski, and Kryukov (2019) develop a related ("BDK") model with no depreciation, but with possible seller entry and exit, to study the existence of predatory equilibria and the efficiency of dynamic competition. We adapt the Besanko, Doraszelski, and Kryukov (2019) decomposition of economic distortions to our setting.

Our primary innovation relative to these dynamic competition papers is to show how buyer bargaining power may affect equilibrium multiplicity, concentration, welfare and counterfactuals. This direction may not have been pursued in the literature because allowing for future negotiations, and bargaining weights, to affect outside options can be seen as complicated.[8] One dynamic competition paper that does allow for bargaining, in this case between Chinese battery and electric vehicle manufacturers, is Barwick, Kwon, Li, and Zahur (2025). When estimating their model, and when

[7] Spence (1981) had performed related numerical comparisons between perfect equilibria and equilibria where firms can precommit.

[8] For example, Aguirregabiria, Collard-Wexler, and Ryan (2021) describe the dynamic game of Lee and Fong (2013), where hospitals and insurers stochastically receive opportunities to negotiate to add the hospital to the insurer's network, as requiring "an enormous level of skill with computation to implement". Papers that consider "bargaining-with-replacement" in static models (e.g., Ho and Lee (2019)) have assumed that, if an agreement breaks down, one of the parties will be able to make a take-it-or-leave-it offer to an alternative counterparty.

performing counterfactuals, which they do without resolving the model, Barwick, Kwon, Li, and Zahur (2025) restrict firms to believe that future and current markups will be equal and unaffected by the outcome of the current bargain. In contrast, we consider a much simpler duopoly model where we focus on how bargaining power, static markups, dynamic incentives and expectations of future market structure interact in equilibrium to affect market outcomes. However, we note that, because we treat τ as a parameter we can vary, we do not address the question of how bargaining weight can be identified and estimated empirically when it affects dynamic incentives.

Bargaining has been studied theoretically in other types of dynamic environments. For example, McAfee and Schwartz (1994), Bedre-Defolie (2012), Dequiedt and Martimort (2015) and Do and Miklós-Thal (2025) study issues of opportunism when a supplier makes agreements with multiple competing retailers, either sequentially or simultaneously. We do not consider competition between our buyers, so the issues examined in these papers do not arise.[9] Houba, Sneek, and Vardy (2000), Sorger (2006) and Flamini (2012) consider bilateral bargaining over a shared asset, such as a fish stock. As in the current paper, bargained outcomes can affect how state variables evolve, but, in our model, two (or more) sellers compete for short-lived buyers through separate bargains, and it is the relative competitive strength of different sellers that is endogenous.

2 Model

The model is an infinite horizon game with discrete time and states. The discount factor is $0 \leq \beta \leq 1$.

Sellers, States and Technology. There are two long-lived ex-ante symmetric but differentiated sellers ($i = 1, 2$). i's production cost is $c(e_i) = \kappa\rho^{\log_2(\min(e_i,m))}$, where $m > 1$, and $e_i = 1, ..., M$ is a commonly observed state variable indicating i's "know-how".[10] Costs therefore fall with know-how only up to level m, but know-how above m can make a firm more resilient to depreciation

[9] One way to rationalize our assumption is that industries we have in mind are often selling capital goods, such as aircraft and rockets, rather than goods that buyers simply re-sell to final consumers.

[10] Asker, Fershtman, Jeon, and Pakes (2020), Sweeting, Roberts, and Gedge (2020) and Sweeting, Tao, and Yao (2024) consider dynamic models where state variables are private information.

shocks. $\rho \in (0,1)$ and lower values of ρ imply stronger learning economies.[11] The industry state is $\mathbf{e} = (e_1, e_2)$. We will describe seller 1 as a know-how "leader" ("laggard") if and only if $e_1 > (<) e_2$.

One of the sellers will sell one unit each period. Except at the state space boundaries,

$$e_{i,t+1} = e_{i,t} + q_{i,t} - f_{i,t} \tag{1}$$

where $q_{i,t}$ is equal to one (zero) if and only if firm i (j) makes the sale, and $f_{i,t}$, the realized depreciation shock, is equal to one (zero otherwise) with probability $\Delta(e_i) = 1 - (1-\delta)^{e_i}$ with $\delta \in [0,1)$.[12] Δ increases in both δ and e_i.

Buyers. Every period a buyer arrives who will purchase exactly one unit from one of the sellers. The buyer is in the market only once, and she chooses the seller that maximizes her indirect utility, $v - p_i + \sigma\varepsilon_i$, where p_i is the price paid to the chosen seller, and the ε_is are i.i.d. private information Type I extreme value payoff shocks. $0 < \sigma < \infty$ determines the degree of seller product differentiation.

Bargaining. BDKS assume sellers make simultaneous take-it-or-leave-it price offers. Instead, we assume a form of "Nash-in-Nash" bargaining. Suppose that, before the εs are realized, the buyer sends separate agents to each seller. Each buyer agent-seller pair negotiates, in a Nash bargain where the buyer bargaining weight is τ (seller weight $1-\tau$), a price at which the buyer can purchase from that seller, taking the price negotiated in the other negotiation as given.[13] Once negotiations are complete, the buyer observes the εs and chooses which seller to buy from, with a purchase possible only if a price was agreed. Choice probabilities are, therefore, given by the standard logit formula given the agreed prices. If $\tau = 1$, the buyer has "all of the bargaining power" and extracts all of the expected surplus from an agreement. BDKS's price-setting assumption is replicated if $\tau = 0$ (sellers have all of the bargaining power).

[11] We assume that $\rho < 1$ so that costs differ across at least some know-how states.

[12] When $e_{i,t} = 1$ and $q_{i,t} = 0$, $e_{i,t+1} = 1$, and when $e_{i,t} = M$ and $q_{i,t} = 1$, $e_{i,t+1} = M$.

[13] The distinction from many other applications is that the agreement of a price does not mean that a unit will necessarily be traded. In our model, agreement provides the buyer with an option to buy.

Parameters. We will primarily use two different specifications. We will show a number of propositions for an $m = M = 2$ ("$M = 2$") model for all $\rho \in (0, 1)$, $\delta \in [0, 1)$, $0 \leq \beta < 1$, $\kappa > 0$, $\sigma > 0$ and v parameters. In fact, our analytical results will not rely on the assumed functional forms for costs or depreciation, i.e., we only need to assume that $c(1) > c(2)$ and that either $\Delta(2) > \Delta(1)$ or $\Delta(2) = \Delta(1) = 0$, although they do assume the logit form of demand with no outside good.

We will use BDKS's "$M = 30$" specification with $m = 15$, $M = 30$, $\beta = \frac{1}{1.05}$, $\sigma = 1$, $\kappa = 10$ and $v = 10$ to analyze a more realistic, though still stylized, model.[14] As there is no outside option, the value of v does not affect the buyer's choice probabilities, and we will exclude it from our surplus calculations. The parameters that we will vary in our $M = 30$ model analysis are therefore the bargaining weight τ, and the technology parameters ρ and δ.

We limit the computational burden by focusing on technologies where $0.6 \leq \rho < 1$ and $0 \leq \delta \leq 0.2$.[15] These ranges cover parameters we believe are empirically plausible. For example, $0.6 \leq \rho < 1$ covers all of the estimated progress ratios from the 97 empirical studies surveyed by Ghemawat (1985). When we refer to technologies where "LBD is significant", we will be referring to values of ρ less than 0.85, accounting for two-thirds of Ghemawat's studies. When we assume $m = 15$, $\rho \leq 0.85$ implies that LBD can lower costs by around 50% or more.

$0 < \delta \leq 0.2$ covers the range from no depreciation, as assumed by Cabral and Riordan (1994), to almost certain depreciation once know-how is high.[16] We will say that depreciation is "not too likely" if $\delta < 0.03$, but the reader should recognize that this language is quite conservative in the sense that $\delta = 0.03$ implies that the probability of depreciation can be quite high when a firm is at the bottom of its cost curve (e.g., $\Delta(20) = 0.456$).[17]

While $\tau = 0$ is rationalized by sellers simultaneously making take-it-or-leave-it offers, it is

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[14] As BDKS discuss, one can interpret this discount factor as corresponding to a period length of 1 month, a monthly discount rate of 1 percent and a probability that the industry survives each month of 0.96.

[15] We have explored equilibria outside this region, and not found results that would change any qualitative point we make below.

[16] For example, if $e_i = 20$ and $\delta = 0.2$, $\Delta(e_i) = 0.989$.

[17] One feature of $\delta < 0.03$ is that both firms can be expected to get to and then remain at the bottom of their cost curves if they split sales equally. Benkard (2000) estimated that 4% of know-how depreciates each month in airframe production, although because multiple units may be sold in a month, the values of "know-how" in Benkard's analysis are much larger than in our model.

less clear which values of τ are most relevant if buyers can bargain. $\tau = 0.5$ is often assumed in applications (e.g., Gowrisankaran, Nevo, and Town (2015)), but, when using static models, researchers have found that a range of weights may be required to rationalize bargained outcomes (Grennan (2013)). We will emphasize some results that reflect what happens as τ increases from zero (i.e., as we relax the price-setting assumption) and other results that reflect differences in outcomes across a wide range of τ values. When we say buyers have "moderate bargaining power", we will be referring to τs between 0.2 and 0.5.[18]

3 Equilibrium: Formulation and Multiplicity

In this section, we provide two characterizations of symmetric Markov Perfect Nash equilibria and discuss when equilibria are unique.

3.1 Formulations of Equations Characterizing Equilibria

Equilibrium Concept: Markov Perfect Nash Equilibrium. Following Cabral and Riordan (1994) and BDKS, the equilibrium concept is symmetric and stationary Markov Perfect Nash equilibrium (MPNE, Maskin and Tirole (2001), Ericson and Pakes (1995), Pakes and McGuire (1994)). A symmetric MPNE will comprise internally consistent prices that solve the bargaining problem, together with state-specific values, given the assumed bargaining weight and technology and taste parameters. We provide two sets of equations that are *necessary and sufficient* for a symmetric MPNE, and equivalent in the sense that the prices and values or choice probabilities implied by a solution to one set of equations will also solve the other set. As with BDKS's model, the existence of at least one symmetric equilibrium in pure strategies follows from a minor extension of Proposition 2 of Doraszelski and Satterthwaite (2010).

[18] If one interprets our myopic buyers as also being short-lived and impatient, then one might view τs close to zero as being more reasonable. However, the myopia assumption does not need to be interpreted as meaning that buyers are less patient than sellers when negotiating prices. For example, a large corporation that only buys a complex and durable machine very infrequently may ignore how its purchase affects future competition between machine producers (the key implication of myopia in our paper), but it may be willing to negotiate patiently for a lower price.

Price and Value Formulation of the Equilibrium Equations. We can formulate a symmetric MPNE in terms of equations, in each state, for seller 1's beginning-of-period equilibrium value ($V_1^{S*}(\mathbf{e})$) and a first-order condition for seller 1's price ($p_1^*(\mathbf{e})$).[19] For values,

$$V_1^{S*}(\mathbf{e}) - D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - c(e_1)) - \sum_{k=1,2} D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e}) = 0, \tag{2}$$

where $D_k^*(\mathbf{e})$ is the equilibrium probability seller k makes the sale, and $\mu_{1,k}^S(\mathbf{e})$ is seller 1's continuation value when seller k makes the sale,

$$\mu_{1,k}^S(\mathbf{e}) = \beta \sum_{\forall e'_{1,t+1}|e_{1,t}} \sum_{\forall e'_{2,t+1}|e_{2,t}} V_1^{S*}(e'_{1,t+1}, e'_{2,t+1}) \Pr(e'_{1,t+1}|e_{1,t}, k) \Pr(e'_{2,t+1}|e_{2,t}, k). \tag{3}$$

$\Pr(e'_{i,t+1}|e_{i,t}, k)$ is the probability that i's state transitions from $e_{i,t}$ to $e'_{i,t+1}$ when $q_{k,t} = 1$.

Given any pair of prices, the probability that an atomistic buyer chooses seller 1 is

$$D_1(p_1, p_2) = \frac{\exp\left(\frac{v-p_1}{\sigma}\right)}{\exp\left(\frac{v-p_1}{\sigma}\right) + \exp\left(\frac{v-p_2}{\sigma}\right)} = \frac{1}{1 + \exp\left(\frac{p_1-p_2}{\sigma}\right)}. \tag{4}$$

Given p_2, continuation and production costs, Nash-in-Nash bargaining implies that $p_1(p_2, \mathbf{e})$ will be determined as

$$\begin{gathered} p_1(p_2, \mathbf{e}) = \arg\max_{p_1} \left[CS(p_1, p_2) - CS(p_2)\right]^{\tau} \times ... \\ \left[D_1(p_1, p_2)(\mu_{1,1}^S(\mathbf{e}) + p_1 - c(e_1)) + (1 - D_1(p_1, p_2))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right]^{(1-\tau)} \end{gathered} \tag{5}$$

where $CS(p_1, p_2) = \sigma \log\left(\sum_{k=1,2} \exp\left(\frac{v-p_k}{\sigma}\right)\right)$, i.e., the expected surplus of the buyer when it is able to choose from both sellers. $CS(p_2) = v - p_2$ is the buyer's expected surplus if there is no agreement with seller 1, so that seller 2 is the buyer's only option. Given an equilibrium p_2^* and continuation values, equilibrium $p_1^*(\mathbf{e})$ will therefore solve the first-order condition

$$\begin{gathered} \tau \frac{\partial CS(p_1^*(\mathbf{e}), p_2^*)}{\partial p_1} \left[D_1^*(\mathbf{e})(\mu_{1,1}^S(\mathbf{e}) + p_1^*(\mathbf{e}) - c(e_1)) + (1 - D_1^*(\mathbf{e}))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right] + ... \\ (1-\tau)\left[CS(p_1^*(\mathbf{e}), p_2^*) - CS(p_2^*)\right] \left[D_1^*(\mathbf{e}) + \frac{\partial D_1^*(\mathbf{e})}{\partial p_1}\left(p_1^*(\mathbf{e}) - c(e_1) + \mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right)\right] = 0, \end{gathered} \tag{6}$$

[19] Symmetry implies that we only need to define equations for the prices and values of seller 1, i.e., $p_2^*(e_1, e_2) = p_1^*(e_2, e_1)$ and $V_2^{S*}(e_1, e_2) = V_1^{S*}(e_2, e_1)$.

where $\frac{\partial CS(p_1^*(\mathbf{e}),p_2^*)}{\partial p_1} = -D_1^*(\mathbf{e})$ and $\frac{\partial D_1^*(\mathbf{e})}{\partial p_1} = -\frac{D_1^*(\mathbf{e})(1-D_1^*(\mathbf{e}))}{\sigma}$. This simplifies to

$$-\tau D_1^*(\mathbf{e})\left[p_1^*(\mathbf{e}) - \widehat{c_1}(\mathbf{e})\right] + (1-\tau)\left[\sigma - (1-D_1^*(\mathbf{e}))(p_1^*(\mathbf{e}) - \widehat{c_1}(\mathbf{e}))\right]\log\frac{1}{1-D_1^*(\mathbf{e})} = 0. \tag{7}$$

$\widehat{c_1}(\mathbf{e}) = c(e_1) - (\mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}))$ is seller 1's economic opportunity cost of a sale, which reflects both the production cost and seller 1's dynamic incentive to make a sale. The first-order condition is identical to BDKS if $\tau = 0$.[20]

Solving the Price and Value Equations and Enumerating Equilibria. Appendix A describes how we numerically solve for equilibria in the $M = 30$ model. A single equilibrium can be found by either solving the equations simultaneously, using the Pakes and McGuire (1994) algorithm or, if $\delta = 0$, using backwards induction. For given τ, we use BDKS's numerical homotopy method to repeatedly criss-cross the (ρ, δ) parameter space to search for additional equilibria. We use homotopies where we change τ when we want to track what happens, for a given technology, as buyer bargaining power varies continuously.

There is no guarantee that homotopies will find all equilibria in this model, and, in practice, homotopies may stop or "stall" (taking a seemingly unending sequence of tiny steps). Therefore, we acknowledge that we may have missed equilibria for any particular set of parameters. However, we are confident in our qualitative results based on homotopies for at least two reasons. First, they are robust to conducting our search in different ways or also searching using τ-homotopies. Second, as discussed in Deng, Jia, Leccese, and Sweeting (2025), an alternative method can be used to find all equilibria when $M = 3$, and this produces identical sets of equilibria to homotopies.

Buyer Choice Probability Formulation of the Equilibrium Equations. The equilibrium probability seller 1 is chosen can be written as

$$\sigma \log\left(\frac{1}{D_1^*(\mathbf{e})} - 1\right) - p_1^*(\mathbf{e}) + p_2^*(\mathbf{e}) = 0. \tag{8}$$

[20] The first-order condition is monotonic in p_1, implying that the second-order condition will also be satisfied. This is easiest to see from the re-written first-order condition (9).

We now show that prices can be expressed as functions of the $D_1^*(\mathbf{e})$s and parameters only.

From the first-order condition for the negotiated price (7), firm 1's markup over its opportunity cost is

$$p_1^*(\mathbf{e}) - \widehat{c}_1(\mathbf{e}) = \Phi(D_1^*(\mathbf{e}), \tau) = \frac{(1-\tau)\sigma \log \frac{1}{1-D_1^*(\mathbf{e})}}{\tau D_1^*(\mathbf{e}) + (1-\tau)(1-D_1^*(\mathbf{e})) \log \frac{1}{1-D_1^*(\mathbf{e})}}. \tag{9}$$

Given $\widehat{c}$s, these markups will be unique and monotonically decreasing in τ (see Lemma 1 and Proposition 2 below). Denoting the seller 1 state transition matrix when k makes a sale as $\mathbf{Q_k}$ and the stacked markup vector as $\mathbf{\Phi}(\mathbf{D_1})$,

$$\widehat{\mathbf{c_1}} = \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})\mathbf{V_1^S}, \tag{10}$$

and

$$\mathbf{V_1^S} = (\mathbf{I} - \beta \mathbf{Q_2})^{-1} [\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{11}$$

where $\circ$ denotes the element-wise product of two vectors. Therefore,

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{12}$$

which can be substituted into equation (8), giving equations in D_1^* only.

We can further reduce the number of equations because the assumptions of symmetry and no outside good imply $D_1^*(e,e) = \frac{1}{2}$, $\forall e$, and $D_1^*(e,e') = 1 - D_1^*(e',e)$. Therefore there are $\frac{M(M-1)}{2}$ equations and unknowns. This compares to $2M^2$ equations for prices and seller values in the standard formulation. For $M = 2$, the reduction is from eight to one.

Advantages of the buyer choice probability formulation. The choice probability formulation allows us to prove equilibrium uniqueness and to provide analytical comparative static results for the $M = 2$ model. With larger state spaces, it provides a convenient way to solve for optimal subsidies.[21] However, the choice probability formulation does not necessarily reduce the computational burden of using homotopies because, in our experience, ρ and δ-homotopies stop or stall more often when

[21] In Deng, Jia, Leccese, and Sweeting (2025), we also use this characterization to employ non-homotopy methods to find all equilibria when $M = 3$. This provides a novel confirmation of homotopy results.

we use the buyer choice probability equations.[22] Our homotopy results use the price and value formulation of the equations.

3.2 Multiple Equilibria

The existence of multiple equilibria illustrates how participants' expectations can play a key role in how an industry evolves. However, multiple equilibria also complicate estimation and undermine the researcher's ability to make clear statements about comparative statics or counterfactuals.[23] Therefore, a researcher is likely to want to know whether allowing for bargaining will either exacerbate or relieve the multiplicity problem that has been documented when sellers set prices. We present analytical results for when equilibria are unique, before using numerical analysis to show that equilibria are unique in the $M = 30$ model when buyers have at least moderate bargaining power.

Analytical Uniqueness Results. The following lemma and proposition detail when uniqueness can be proven analytically. All proofs are in Appendix B.

Lemma 1 *Consider a state* (e_1, e_2) *and hold fixed seller opportunity costs,* $\widehat{c}_1$ *and* $\widehat{c}_2$*. For all* τ*, equilibrium prices are unique.*

This lemma, which implies that unique (p_1^*, p_2^*) will satisfy equations (4) and (7), or equivalently (8) and (9), extends BDKS's "statewise uniqueness" property to all values of τ, implying that multiplicity in the full game must come from players' expectations of the future.

Proposition 1 *In the dynamic model there will be a unique symmetric MPNE if*

1. $\delta = 0$*, or*

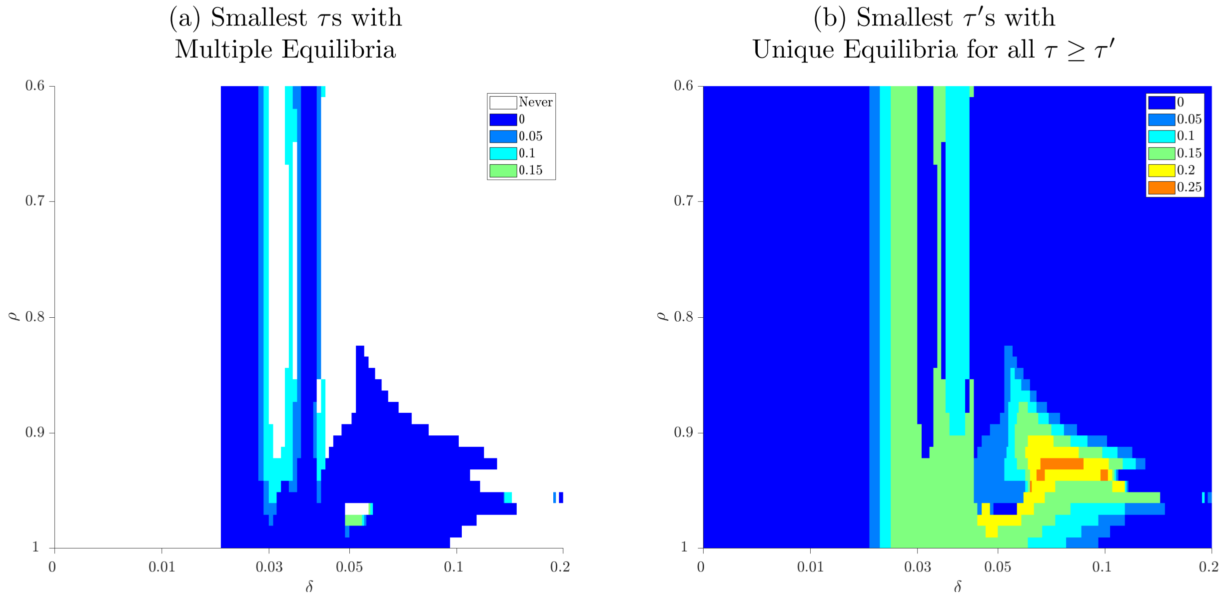
2. $\tau = 1$*, or*

3. $M = 2$.

[22] The obvious explanation for this pattern is that the equations (8), with logarithms and inverses, are more non-linear in the technology parameters than the standard equations.

[23] For example, BDKS, p. 456, discuss how "the multiplicity problem ... plagues the estimation of dynamic stochastic games and inhibits the use of counterfactuals in policy analysis."

Figure 1: Equilibrium Multiplicity in the $M = 30$ Specification Based on Homotopy Analysis for τ in 0.05 Increments.



Uniqueness when know-how cannot depreciate, as assumed in much of the theoretical literature, reflects the directional game logic of Iskhakov, Rust, and Schjerning (2016). When $\tau = 1$, the only equilibrium will have prices equal to production costs in all states. A less intuitive, but very useful, result is that there is a unique symmetric equilibrium when there are just two know-how states for each seller.

Numerical evidence on multiplicity and uniqueness in the $M = 30$ Specification. We search for equilibria by criss-crossing the parameter space using ρ and δ-homotopies for values of τ on a grid with 0.05 spacing. We will use the equilibria that we find several times in the analysis below.

Figure 1(a) shows, for a fine grid of (ρ, δ)s, the smallest τs for which we identify multiplicity. We use a log scale for δ, so more space is given to smaller values of δ. Equilibria are always unique in the unshaded region. While there are some technologies for which multiplicity exists when $\tau = 0.05$, 0.1 or 0.15, but not when $\tau = 0$, these account for a small proportion of the technologies in the picture and they are adjacent to technologies where price-setting supports multiplicity.[24]

[24]For $\tau = 0$, we find almost identical numbers of equilibria to BDKS, except that we identify some additional

Panel (b) shows the smallest values of τ for which equilibria are unique for all weakly larger τs on our grid. We find that equilibria are unique for all technologies if $\tau \geq 0.25$. Compared to $\tau = 0$, the proportion of technologies with multiple equilibria is much smaller when $\tau > 0.1$.[25]

The fact that we identify unique equilibria once buyers have moderate bargaining power will help us to summarize comparative statics. While we cannot be sure what happens in dynamic games with different specifications, our results suggest that, in at least some situations, allowing for buyer bargaining power may not only make models more realistic, but also alleviate one of the problems that has made working with dynamic games difficult.

4 Equilibrium Incentives, Market Structure and Efficiency

In this section we focus on how buyer bargaining power and dynamics affect equilibrium outcomes, focusing first on the effects of buyer bargaining power in the fully dynamic model, before contrasting what we find with a model where sellers are static.

4.1 Market Structure and the Difference in the Leader and Laggard's Dynamic Incentives

A classic question in the theoretical dynamic competition literature is whether equilibrium behavior will tend to lead to an industry being dominated by a single firm (e.g., Fudenberg and Tirole (1983), Budd, Harris, and Vickers (1993), Cabral and Riordan (1994), Athey and Schmutzler (2001), BDKS). It is therefore natural to start our investigation by asking how buyer bargaining power affects market structure.

Analytical results. Single-firm dominance will result if know-how leaders are much more likely to make sales in asymmetric states. Consider any asymmetric state where $e_1 < e_2$ (i.e., seller 2 is the know-how leader). The leader sells with probability

equilibria for a very small range of parameters where ρ is very close to 1.

[25] We have also run homotopies when $\tau = 0.01$ and $\tau = 0.025$, and we also see reductions in set of technologies supporting multiplicity for these cases, as well as reductions in the number of equilibria where multiplicity exists. Deng, Jia, Leccese, and Sweeting (2025) show that we also find multiplicity only if $\tau < 0.25$ in a model where $m = M = 3$, using both homotopy and non-homotopy methods to identify equilibria.

$$D_2^*(e_1, e_2) = \frac{1}{1 + \exp\left(\frac{p_2^*(\mathbf{e}) - p_1^*(\mathbf{e})}{\sigma}\right)}. \tag{13}$$

$D_2^*(e_1, e_2)$ therefore varies monotonically with the difference in equilibrium prices. We can decompose this price gap as

$$p_2^*(\mathbf{e}) - p_1^*(\mathbf{e}) = \underbrace{c_2 - c_1}_{\substack{\text{Difference in} \\ \text{Costs}}} + \underbrace{[\Phi(D_2^*(\mathbf{e}), \tau) - \Phi(D_1^*(\mathbf{e}), \tau)]}_{\text{Difference in Static Markups}} - \underbrace{[DI_2(\mathbf{e}, \tau) - DI_1(\mathbf{e}, \tau)]}_{\text{Difference in Dynamic Incentives}} \tag{14}$$

where the static markup, $\Phi(D_k^*(\mathbf{e}), \tau)$, was defined in equation (9). $DI_k(\mathbf{e}) = \mu^S_{k,k}(\mathbf{e}) - \mu^S_{k,-k}(\mathbf{e})$ is seller k's "dynamic incentive", i.e., the difference between k's continuation values when k, rather than its rival, makes the sale. $DI_2(\mathbf{e}, \tau) - DI_1(\mathbf{e}, \tau)$ is therefore the difference in the dynamic incentives of the leader and the laggard, which we will call the DDI in what follows. A larger DDI will favor the leader winning more often. If $\tau = 1$, the DDI will equal zero.

As τ increases, changes in the leader's sale probability will reflect how the buyer's increased bargaining weight affects the differences between the static markups and the DDI. The following proposition shows that, when we hold the DDI fixed, an increase in τ will shrink the markup of the firm which has the lower opportunity cost when $\tau = 0$, implying that it is more likely to make the sale when sellers set prices, more than its rival's markup. As a result, the probability that this firm wins will further increase.

Proposition 2 *Consider a one-shot bargaining game where sellers 1 and 2 have fixed opportunity costs of sale, $c_1 - DI_1$ and $c_2 - DI_2$ respectively. The following three properties hold for all technology and taste parameters:*

1. *if and only if $c_i - DI_i < c_j - DI_j$, then $D_i^* > \frac{1}{2}$ for all τ.*

2. *the static markups, $\Phi(D_k^*(\mathbf{e}), \tau)$, and therefore the prices, of both firms strictly decrease in τ.*

3. *if $c_i - DI_i < c_j - DI_j$, then, for any increase in τ, the static markup of firm i decreases more than the markup of firm j, and D_i^* is strictly increasing in τ.*

When $M = 2$, there is a single asymmetric state, and the following proposition shows that, in that state, the know-how leader will be more likely to win when $\tau = 0$. Therefore, the laggard can only become more likely to win as τ increases if its dynamic incentive increases relative to the dynamic incentive of the leader. However, the proposition also shows that it is the leader's dynamic incentive which will grow, relative to the laggard's, when $\tau = 0$, and that even if this pattern reverses for higher τ, the changes in the DDI will never be enough to prevent the probability that the leader wins increasing monotonically in τ.

Proposition 3 *Consider the model where $M = 2$. In state $\mathbf{e} = (1, 2)$,*

1. *the derivative of $DI_2(\mathbf{e}, \tau) - DI_1(\mathbf{e}, \tau)$ with respect to τ, evaluated at $\tau = 0$, is greater than 0.*
2. *$D_2^*(\mathbf{e}) > \frac{1}{2}$ for all τ, and $D_2^*(\mathbf{e})$ monotonically increases in τ.*
3. *the probability that the industry is in state (1,2) from the third period onwards is increasing in τ.*

The property that the DDI always increases in τ when τ is small is not trivial, as two forces may tend to offset each other. If winning the current sale increases the leader's expected future profits more than the laggard, then, because an increase in τ implies that the a seller will get a smaller share of surplus in future negotiations, the leader's dynamic incentive may tend to fall relative to the dynamic incentive of the laggard when the distribution of future states is held fixed.

However, a second factor, which we call the "lead lengthening effect", can increase the leader's relative dynamic incentive. As the static markup effect increases D_2^*(1,2), a leader will expect its lead to last longer if it makes the current sale. This will tend to affect the current leader's dynamic incentive more than the current laggard's because, when it sells, the current leader will preserve its lead for sure, whereas the laggard will only attain the lead with lower probability (zero probability if $\delta = 0$). Appendix C.1 illustrates how static markups, dynamic incentives, the leader's sale probability and how long the lead is expected to last vary with τ for two different technology parameters.

Dynamic incentives and buyer bargaining power in the $M = 30$ specification: Numerical results. The $M = 30$ specification is not analytically tractable because changes in τ may move equilibrium play across states where the levels and changes in static markups and dynamic incentives may vary. However, we can compute how summary present value measures of dynamic incentives change with τ, assuming equilibrium play and a game starting in (1,1). Our summary measure of leader dynamic incentives is

$$\text{Leader } DI^{PDV} = \sum_{t=2}^{\infty} \beta^{t-1} \sum_{\mathbf{e} \in e_1 \neq e_2} \frac{\lambda_t(\mathbf{e})}{1 - \sum_{\mathbf{e}^\dagger \in e_1^\dagger = e_2^\dagger} \lambda_t(\mathbf{e}^\dagger)} \mathbb{I}(e_2 > e_1) DI_2(\mathbf{e}), \tag{15}$$

and our laggard summary measure is

$$\text{Laggard } DI^{PDV} = \sum_{t=2}^{\infty} \beta^{t-1} \sum_{\mathbf{e} \in e_1 \neq e_2} \frac{\lambda_t(\mathbf{e})}{1 - \sum_{\mathbf{e}^\dagger \in e_1^\dagger = e_2^\dagger} \lambda_t(\mathbf{e}^\dagger)} \mathbb{I}(e_2 > e_1) DI_1(\mathbf{e}), \tag{16}$$

where $\lambda^t(\mathbf{e})$ is the probability that the state will be $\mathbf{e}$ in the t^{th} period. We start the summation in $t = 2$ as the firms are symmetric when the game starts.

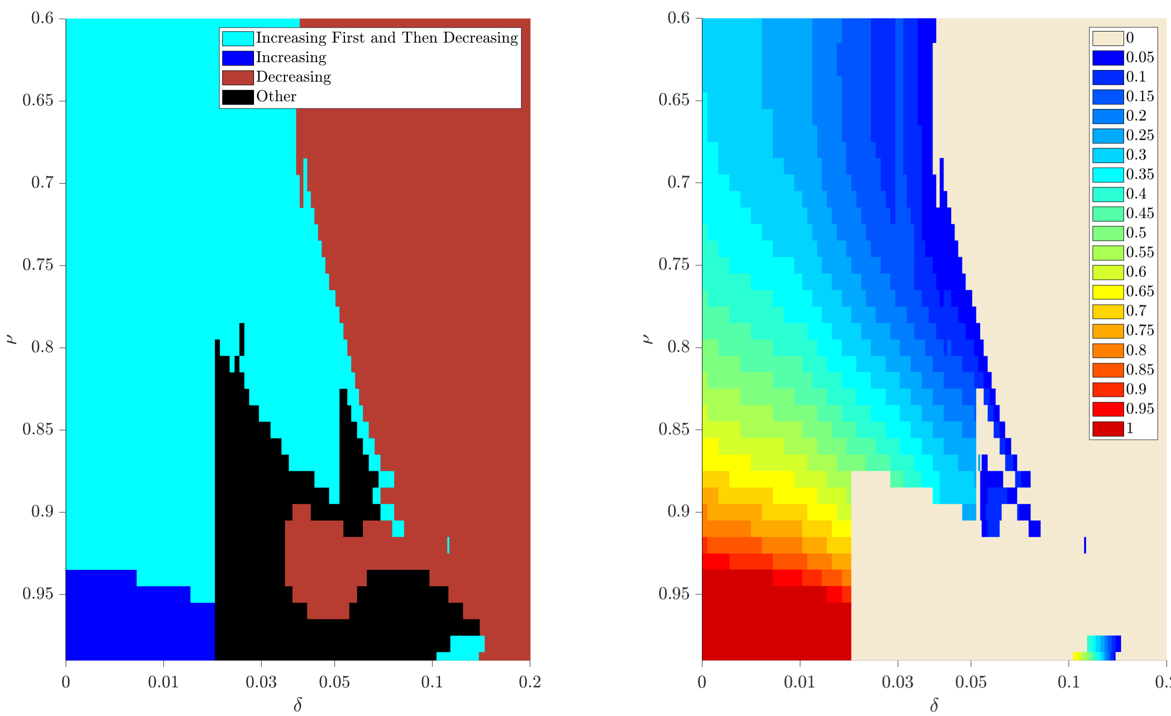
Figure 2(a) shows technology parameters for which the Leader DI^{PDV} $-$Laggard DI^{PDV} statistic is either monotonically increasing, monotonically decreasing, or "monotonically increasing-then-monotonically decreasing" with respect to τ when we use the equilibria identified for 0.05 steps of τ from zero to one. We use the maximum value of the difference when there are multiple equilibria, and most of the technologies where the changes do not fall into one of these three categories ("Other") have multiplicity. Panel (b) shows the value of τ that maximizes the difference between the leader and laggard dynamic incentives. We will continue to use the maximum value of various statistics in later figures: the reader can consult Appendix C.3 to see figures that use the minimum value of the statistic when there are multiple equilibria. These figures support similar conclusions.

When depreciation is not too likely, the dominant pattern is that the leader's dynamic incentives become larger, relative to those of the laggard, as τ increases, at least until buyers have moderate bargaining power. Looking ahead, Figure 3(a) shows the expected value of a Herfindahl concen-

Figure 2: Monotonicity of the Difference Between the PDVs of Leader and Laggard Dynamic Incentives for the $M = 30$ Specification for Different Technologies. We take the maximum value of the statistic across equilibria when multiple equilibria exist for given τ.

(a) Monotonicity of Leader DI^{PDV} – Laggard DI^{PDV}

(b) Value of τ Maximizing Leader DI^{PDV} – Laggard DI^{PDV}

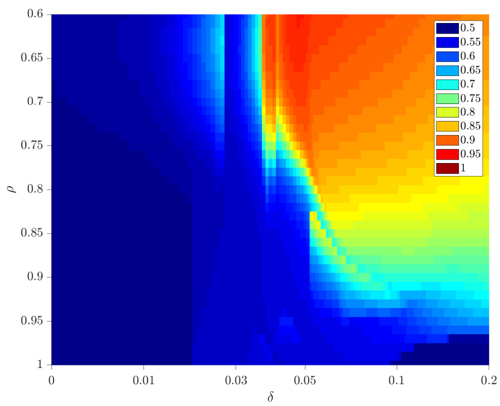


tration measure after 32 periods when $\tau = 0$. Concentration when $\tau = 0$ is low if depreciation is not too likely, so that leads may not be expected to last long (as confirmation of this connection, Appendix Figure C.2(a) shows how many periods, on average, a leader in period 32 would expect its lead to last), so that there is considerable scope for the lead lengthening effect to increase the leader's dynamic incentives. On the other hand, when depreciation is much more likely, leads are likely to last a long time in the $\tau = 0$ equilibrium, so that the larger effect of increasing buyer bargaining power is through shrinkage of the leader's continuation values, which will cause the leader's dynamic incentives to shrink relative to those of the laggard (red color in panel (a) and the beige color in panel (b)).[26]

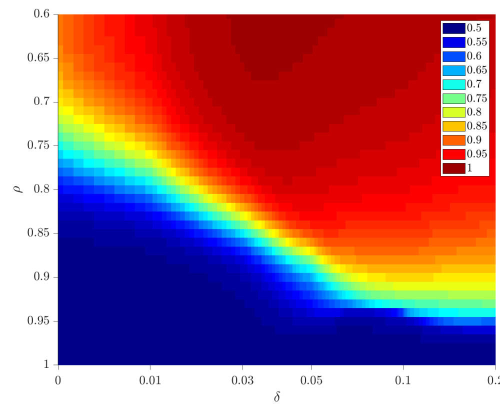
[26] In the $M = 2$ model, the laggard is potentially only one sale away from taking the lead, so the leader cannot attain the same type of entrenched dominance as in the $M = 30$ specification. Appendix Figure C.2(b) shows what happens to the expected lead length as τ increases. If depreciation is not too likely, the expected lead length increases monotonically in τ. When depreciation is more likely, there is more heterogeneity in the relationship,

Figure 3: Expected Value of the HHI^{32} in the $M = 30$ Specification in Dynamic Equilibrium for Alternative Values of τ and in the Social Planner Solution. HHI^{32} value is the maximum across equilibria if multiple equilibria are identified.

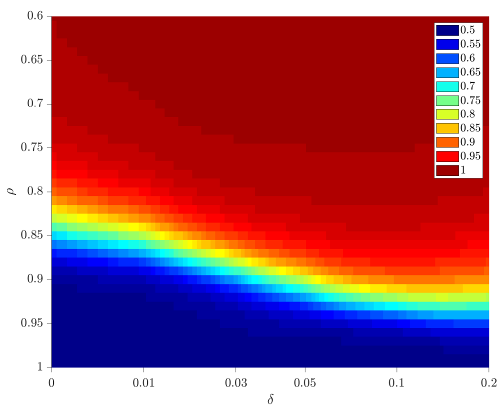
(a) $\tau = 0$



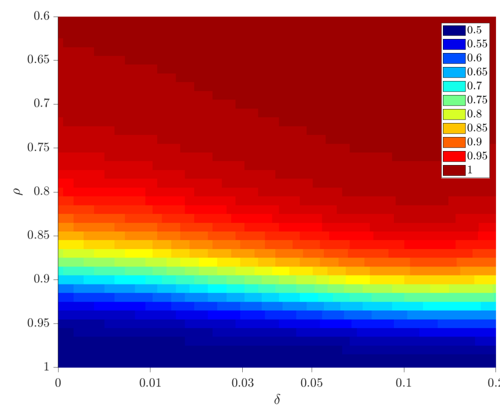
(b) $\tau = 0.25$



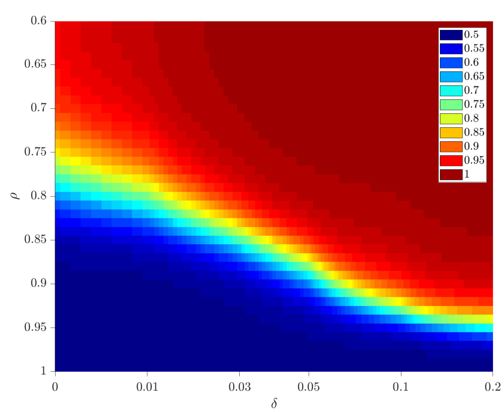
(c) $\tau = 0.5$



(d) $\tau = 1$



(e) Social Planner



Market structure and buyer bargaining power when $M = 30$: Numerical results. We now show how τ affects a summary Herfindahl measure of equilibrium market structure. For a specific period t

$$HHI^t = \sum_{\forall \mathbf{e}} \lambda^t(\mathbf{e}) HHI(\mathbf{e}) \text{ where } HHI(\mathbf{e}) = \sum_{k=1,2} \left(\frac{D_k^*(\mathbf{e})}{D_1^*(\mathbf{e}) + D_2^*(\mathbf{e})} \right)^2.$$

The minimum HHI^t is 0.5. In the $M = 2$ model, Proposition 3 implies HHI^t increases in τ for all $t > 2$. For the $M = 30$ specification, we will focus on HHI^{32}. The choice of thirty-two periods is arbitrary, but it is long enough such that, with no depreciation, both sellers could reach the flat sections of their cost curves if they split sales equally.[27] We see broadly similar patterns for other values of t, such as $t = 200$.

Panel (a) of Figure 3 shows the level of HHI^{32} (maximum value if there are multiple equilibria) when $\tau = 0$. As already noted, concentration tends to be high when depreciation is likely, unless ρ is close to 1 so that LBD has little effect on costs.

Panels (b)-(d) show the expected levels of HHI^{32} when $\tau = 0.25$, 0.5 or $\tau = 1$ (all equilibria are unique for these parameters). When LBD is significant ($\rho < 0.85$) and depreciation is not too likely ($\delta < 0.03$), concentration rises quite dramatically, from close to symmetry to close to monopoly, when τ increases from 0 to 0.5, which is, of course, consistent with the leader's dynamic incentives increasing relative to those of the laggard and the changes in static markups discussed above.

The reader may notice that there are technologies where the reported HHI^{32} falls as τ increases. Most of these technologies support multiple equilibria when $\tau = 0$, and some of them reflect our use of the maximum value of HHI^{32}. We also see some small decreases in concentration, although concentration remains at very high levels, when τ increases from 0.5 to 1 and technologies imply significant LBD and moderate depreciation (e.g. $\rho = 0.7$ and $\delta = 0.02$). These examples further

consistent with changes in dynamic incentives being able to favor the laggard winning more some technologies, although increasing or increasing-then-decreasing patterns remain common, and there are no technologies for which lead length monotonically declines in τ.

[27]Readers should understand that HHIs may converge very slowly to long-run values. For example, if $\rho = 0.75$ and $\delta = 0.023$ in the $M = 30$ specification, $HHI^t \approx 0.5$ for $t > 4,000$, but $HHI^{1,000} \approx 0.6$.

illustrate the role of dynamic incentives in determining market structure, because, in these cases, the difference in dynamic incentives tends to be large and positive when $\tau = 0.5$, favoring the leader making sales, whereas the difference will necessarily fall to zero as τ approaches 1.

4.2 The Effects of Buyer Bargaining Power on Economic Distortions and Efficiency

Another focus of the dynamic competition literature has been the evaluation of whether equilibrium outcomes are efficient (e.g., Fudenberg and Tirole (1983), Cabral and Riordan (1994) and Besanko, Doraszelski, and Kryukov (2019) with LBD, and Halaburda, Jullien, and Yehezkel (2020) and Biglaiser and Crémer (2020) with network effects). The promotion of efficiency may also be a goal when designing counterfactuals. Most relevant for our analysis, Besanko, Doraszelski, and Kryukov (2019) provide a detailed analysis of distortions and efficiency using the BDK model, assuming seller price-setting. Their main conclusion is that dynamic competition tends to lead to relatively small losses of efficiency (e.g., deadweight loss is less than 20% of the value added by the industry in 90 percent of the parameterizations that they consider), even though equilibrium market structures can be quite different from those a social planner would choose, because different distortions often offset each other.

We define our measure of efficiency and two types of distortion. We show that buyer bargaining power often switches the direction of these distortions, and we provide an analytical characterization of when efficiency will be monotonically increasing or increasing-then-decreasing in τ in the $M = 2$ model. We show that moderate buyer bargaining power tends to minimize distortions and maximize efficiency in the $M = 30$ specification when depreciation is not too likely.

Definitions. We define the expected total surplus in any state, $TS(\mathbf{e})$, as the expected value of the ε of the purchased good, given equilibrium prices and choice probabilities, less its production cost. We do not include v in our calculation as, with no outside good, its value does not affect strategies. Our efficiency measure is the discounted value of total surplus when the game starts in (1,1):

$$TS^{PDV} = \sum_{t=0,..,\infty} \beta^t \sum_{\forall \mathbf{e}} \lambda^t(\mathbf{e}) TS(\mathbf{e}) \text{ where } TS(\mathbf{e}) = \sum_{k=1,2} D_k(\mathbf{e})(-\sigma \log D_k(\mathbf{e}) - c_k) \tag{17}$$

where $-\sigma \log D_k(\mathbf{e})$ is the expected value of the ε_k when k is chosen.

We compare equilibrium outcomes with those a social planner would choose. We model the social planner as a forward-looking buyer who chooses which seller to buy from to maximize TS^{PDV}. The social planner's problem can be characterized, using choice probabilities, as follows.

$$D_1^{SP}(\mathbf{e}) = \frac{1}{1 + \exp\left(\frac{c_1(\mathbf{e}) - c_2(\mathbf{e}) + \mu_2^{SP}(\mathbf{e}) - \mu_1^{SP}(\mathbf{e})}{\sigma}\right)} \tag{18}$$

where μ_i^{SP} is the discounted continuation surplus when seller i is chosen. In vector form,

$$\mu_{\mathbf{2}}^{\mathbf{SP}} - \mu_{\mathbf{1}}^{\mathbf{SP}} = \beta(\mathbf{Q_2} - \mathbf{Q_1}) \left(\mathbf{I} - \beta \sum_{k=1,2} \text{diag}\{\mathbf{D_k^{SP}}\}\mathbf{Q_k}\right)^{-1} \sum_{k=1,2} \left[\mathbf{D_k^{SP}} \circ \left(\sigma \log \frac{1}{\mathbf{D_k^{SP}}} - \mathbf{c_k}\right)\right], \tag{19}$$

Substituting (19) into (18) leads to $\frac{M(M-1)}{2}$ equations for $\frac{M(M-1)}{2}$ asymmetric state D_1^{SP}s.

Besanko, Doraszelski, and Kryukov (2019) decompose the difference between equilibrium and social planner efficiency into three distortions. In our model, where there is no entry or exit, and no outside good, the equivalent decomposition has two components.[28] Formally, and using slightly different labels to Besanko, Doraszelski, and Kryukov (2019), $DWL_\beta = TS^{PDV,SP} - TS^{PDV} = DWL_\beta^{CURR} + DWL_\beta^{TRANS}$.[29]

$$DWL_\beta^{CURR} = \sum_{t=1}^{\infty} \beta^t \sum_{\mathbf{e}} \lambda_t(\mathbf{e}) \left[TS^{SP}(\mathbf{e}) - TS(\mathbf{e})\right]$$

measures "current" surplus losses in the states visited in equilibrium due to differences between the atomistic buyer's choice probabilities and the social planner buyer's choice probabilities. DWL_β^{CURR}

[28] Of course, deadweight loss can be decomposed in many different ways: for example, into losses due to an excessive tendency to buy from the leader or the laggard, and losses which occur early in the game and later in the game. We focus on similar distortions to Besanko, Doraszelski, and Kryukov (2019) for simplicity and because buyer bargaining power affects them in ways that are particularly clear.

[29] Besanko, Doraszelski, and Kryukov (2019) label DWL_β^{TRANS} as a market structure distortion, and they label DWL_β^{CURR} as a pricing distortion. While these labels are useful in the context of the BDK model, they are less helpful here as we have defined market structure in terms of choice probabilities, and both distortions reflect different ways in which equilibrium and planner choice probabilities differ.

can be negative because the social planner is maximizing the sum of current and discounted future surplus, not just current surplus. However, Besanko, Doraszelski, and Kryukov (2019) show that the equivalent distortion is positive for 96% of considered parameterizations in the BDK model.

$$DWL_{\beta}^{TRANS} = \sum_{t=1}^{\infty} \beta^t \sum_{\mathbf{e}} \left[\lambda_t^{SP}(\mathbf{e}) - \lambda_t(\mathbf{e})\right] TS^{SP}(\mathbf{e}),$$

measures "transitional" surplus losses, that reflect the social planner choosing a different evolution of the industry.

Buyer bargaining power, the direction of the distortions and efficiency. A simple result follows immediately from the distortion definitions.[30]

Remark 1 *If $\tau = 1$, DWL_{β}^{CURR} will be weakly negative and DWL_{β}^{TRANS} will be weakly positive. The signs will be strict if the $\tau = 1$ equilibrium does not implement the planner solution.*

It is therefore natural to ask what the signs of the distortions are, in our model, if sellers set prices. Figure 4(a) summarizes what these signs are in the $M = 30$ specification. Black shading indicates that the signs differ across equilibria. DWL_{β}^{CURR} is positive for almost all technologies for which a single sign can be identified. For these technologies, DWL_{β}^{CURR} must therefore change sign, at least once, when τ increases. On the other hand, the sign of DWL_{β}^{TRANS} when $\tau = 0$ depends on the likelihood of depreciation. When depreciation is not too likely, DWL_{β}^{TRANS} is typically negative, implying that it must also change sign as τ increases.

Focusing on technologies for which DWL_{β}^{CURR} is positive and DWL_{β}^{TRANS} is negative when $\tau = 0$, Figure 4(b) and (c) show the values of τ where the changes in sign happen. There are only a few technologies, colored in light grey, with multiple sign flips. Both distortions typically change signs for similar moderate levels of buyer bargaining power (e.g. τ between 0.2 and 0.4), implying that equilibria for these values of τ must be almost as efficient as the planner outcome.

[30] The DWL_{β}^{CURR} result follows immediately from the fact that, when $\tau = 1$, prices equal production costs and a myopic buyer will therefore maximize current surplus. The DWL_{β}^{TRANS} result follows from the fact that the sum of DWL_{β}^{CURR} and DWL_{β}^{TRANS} must be weakly positive.

A comparison of panels (b) and (e) of Figure 3 shows that, when depreciation is not too likely, the levels of HHI^{32} in the equilibrium when $\tau = 0.25$ and the social planner solution are also very similar, implying that, at least by this metric, equilibrium play when buyers have moderate bargaining power is leading to a market structure similar to the one that the planner would choose.[31]

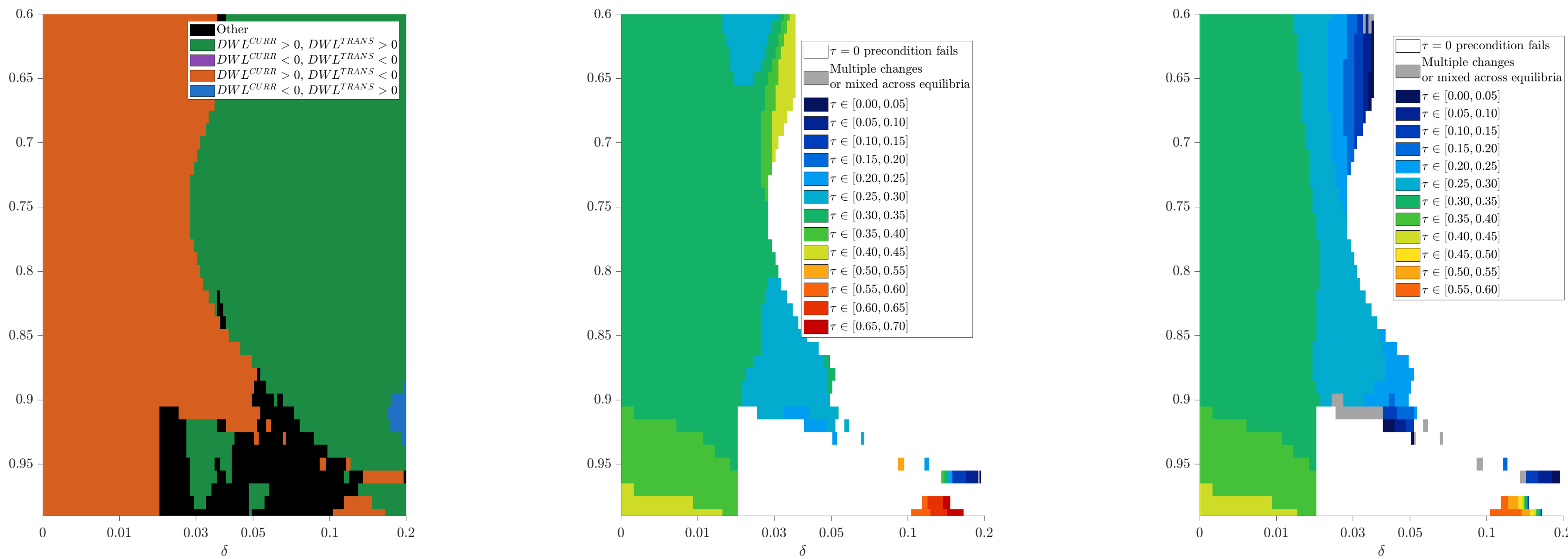
The following proposition characterizes the shape of the τ-efficiency relationship in the $M = 2$ model.

Proposition 4 *In the $M = 2$ model, TS^{PDV} is strictly monotonically increasing in τ if and only if $\frac{c(1)-c(2)}{\sigma} \geq \Lambda(\beta, \Delta(1), \Delta(2))$ where $\Lambda(\beta, \Delta(1), \Delta(2))$ is a function that decreases in the last two arguments. Otherwise, TS^{PDV} is strictly monotonically increasing in τ up to some value $0 < \tau' < 1$, where the equilibrium implements the social planner solution, and then it is strictly monotonically decreasing.*

The proof provides the exact form of the Λ function. Given a value of the discount factor, efficiency monotonically increases in τ if and only if ρ is small enough (i.e., $c(1) - c(2)$ is large enough), product differentiation is sufficiently limited, and the probabilities of depreciation are sufficiently high. These relationships are consistent with our earlier analysis. For example, if product differentiation (variety) is important, or depreciation is unlikely, the social planner will prefer a symmetric market structure with two low cost firms, whereas, in a market equilibrium, the state is likely to be asymmetric when buyers have most of the bargaining power.

[31] It would be interesting to evaluate whether moderate buyer bargaining power would lead to equilibrium market structures being more similar to the social planner market structure in the BDK model. Besanko, Doraszelski, and Kryukov (2019) find that equilibria tend to lead to duopoly more frequently than the planner outcome. In that model, buyer bargaining power would tend to lower seller profits potentially making monopoly more likely.

Figure 4: The Direction of the Distortions when $\tau = 0$, and the Values of τ for which the Distortions Change Sign in the $M = 30$ Specification. In panel (b) the colors indicate the value of τ at which DWL_{β}^{CURR} changes from positive to negative conditional on DWL_{β}^{CURR} being positive and DWL_{β}^{TRANS} being negative when $\tau = 0$. In panel (c) the colors indicate the value of τ at which DWL_{β}^{TRANS} changes from negative to positive with the same precondition.



(a) Signs of Distortions at $\tau = 0$

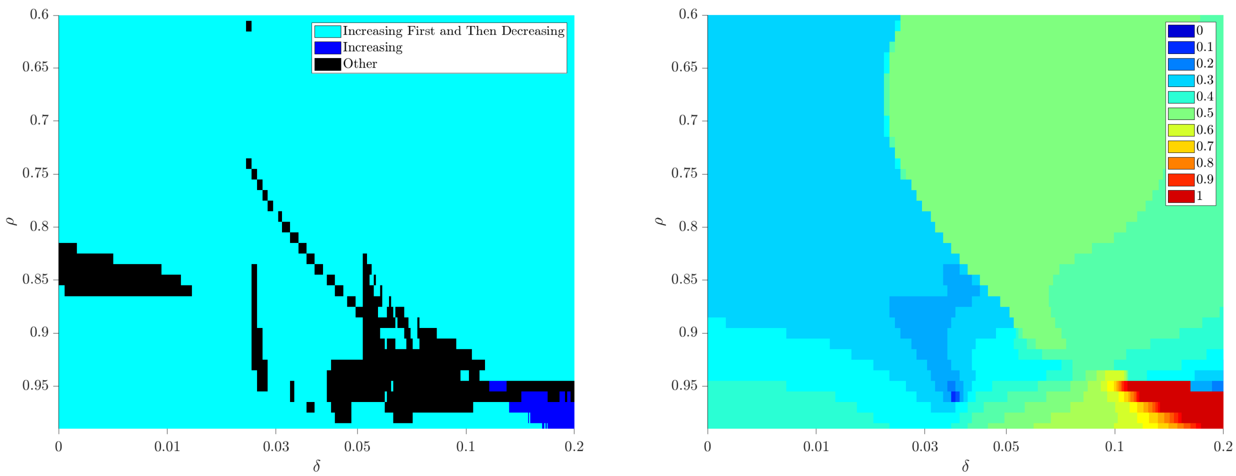
(b) Value of τ Where DWL_{β}^{CURR} Switches from Positive to Negative

(c) Value of τ Where DWL_{β}^{TRANS} Switches from Negative to Positive

Figure 5: Comparative Statics of Efficiency in the $M = 30$ specification for Different Technologies. We take the maximum value of TS^{PDV} across equilibria when multiple equilibria exist for given τ.

(a) Monotonicity of TS^{PDV}

(b) τ Maximizing TS^{PDV}



If efficiency is not monotonically increasing, then it will be increasing-then-decreasing, implying that the probability that the leader sells in state (1,2) is always too low when sellers set prices. The social planner outcome will be implemented exactly when the relationship switches from increasing to decreasing, reflecting that, in the $M = 2$ model, there is only a single choice probability that is relevant and that increases monotonically in τ. However, we have seen that when depreciation is not too likely and buyers have moderate bargaining power, equilibria also come close to attaining social planner efficiency, even if they do not implement the planner solution exactly, in the $M = 30$ specification.

Two further results from the $M = 30$ specification round out our discussion. Figure 5(a) identifies technology parameters where (the maximum value across equilibria of) TS^{PDV} in the $M = 30$ specification is either monotonically increasing-then-monotonically decreasing or monotonically increasing in τ. For the ranges of ρ and δ considered, the typical pattern is hump-shaped.[32] Panel (b) shows the values of τ for which the equilibrium maximizes efficiency for each technology. Consistent with earlier results, efficiency is maximized for moderate buyer bargaining power when

[32] While most of the technologies that cannot be classified (black) have multiple equilibria for small τ, there is also a small non-classified region with low δ and significant LBD (e.g. $\rho = 0.83$), where equilibria are always unique. A closer analysis of examples in this region shows that the increasing-then-decreasing pattern of TS^{PDV} holds until τ is greater than 0.8, but TS^{PDV} increases slightly as τ approaches 1. For all of the unclassified areas, equilibrium efficiency when $\tau = 0.5$ is greater than its value when $\tau = 1$.

depreciation is not too likely. However, the panel also shows that efficiency is maximized for τs that are only slightly higher (e.g. $\tau \approx 0.5$) when depreciation is more likely, as long as LBD has more than a very limited effect on costs.

4.3 Comparison with the Relationships Implied by Static Seller Behavior

We have focused so far on comparing outcomes when sellers set prices and when buyers have some bargaining power in the dynamic model. While we have shown that dynamic incentives and buyer bargaining power interact, a natural question is whether the relationships between buyer bargaining power, market structure and efficiency would be different in a static model. We define what we mean by a model with static seller behavior, before looking at the concentration and efficiency relationships in turn. In Section 5 we provide further analysis of the differences between static and dynamic models in the context of counterfactuals.

Definition of the static seller equilibrium. There are two potential comparisons that one could make. One comparison would be with a model where sellers' costs cannot change, so that MPNE outcomes would be the same as the unique equilibrium in a one-shot bargaining game. If sellers have asymmetric costs, Proposition 2 implies that concentration will increase monotonically in τ, and it is also straightforward to show that efficiency will increase monotonically in τ as the difference in static markups is reduced.

However, our main focus here will be on a comparison to a game where seller know-how can accumulate and depreciate as in the dynamic model, but buyers and sellers treat the sellers' opportunity costs as identical to current production costs, so that sellers' dynamic incentives are ignored.[33] In this model, equilibria will be unique with static markups determined by equations (8) and (9). The static seller and dynamic seller equilibria will coincide if $\tau = 1$. Therefore, we are interested in whether there are differences when sellers have some bargaining power. We note that relative to a model with an outside good, one might expect differences to be smaller in our model

[33]This behavior would be the equilibrium outcome in the dynamic model if sellers' discount factor were set equal to zero, even if the planner's discount factor is set equal to $\frac{1}{1.05}$.

as below cost pricing is not required to start moving sellers down their cost curves.

The effect of buyer bargaining power on market structure. Figure 6(a)-(d) replicate the $M = 30$ concentration heatmaps in Figure 3(a)-(d) except that the new figures assume static seller behavior.

If depreciation is not too likely, the level of concentration is low when $\tau = 0$ given either dynamic or static seller behavior. Therefore, if an analyst tried to evaluate the importance of accounting for sellers' dynamic incentives by comparing concentration when $\tau = 0$, they might conclude that accounting for them is not so important.[34] However, if LBD is significant, there are large differences in expected concentration when τ is 0.25 or 0.5, with concentration being much higher when sellers are dynamic, reflecting how moderate buyer bargaining power increases leaders' dynamic incentives relative to those of laggards.

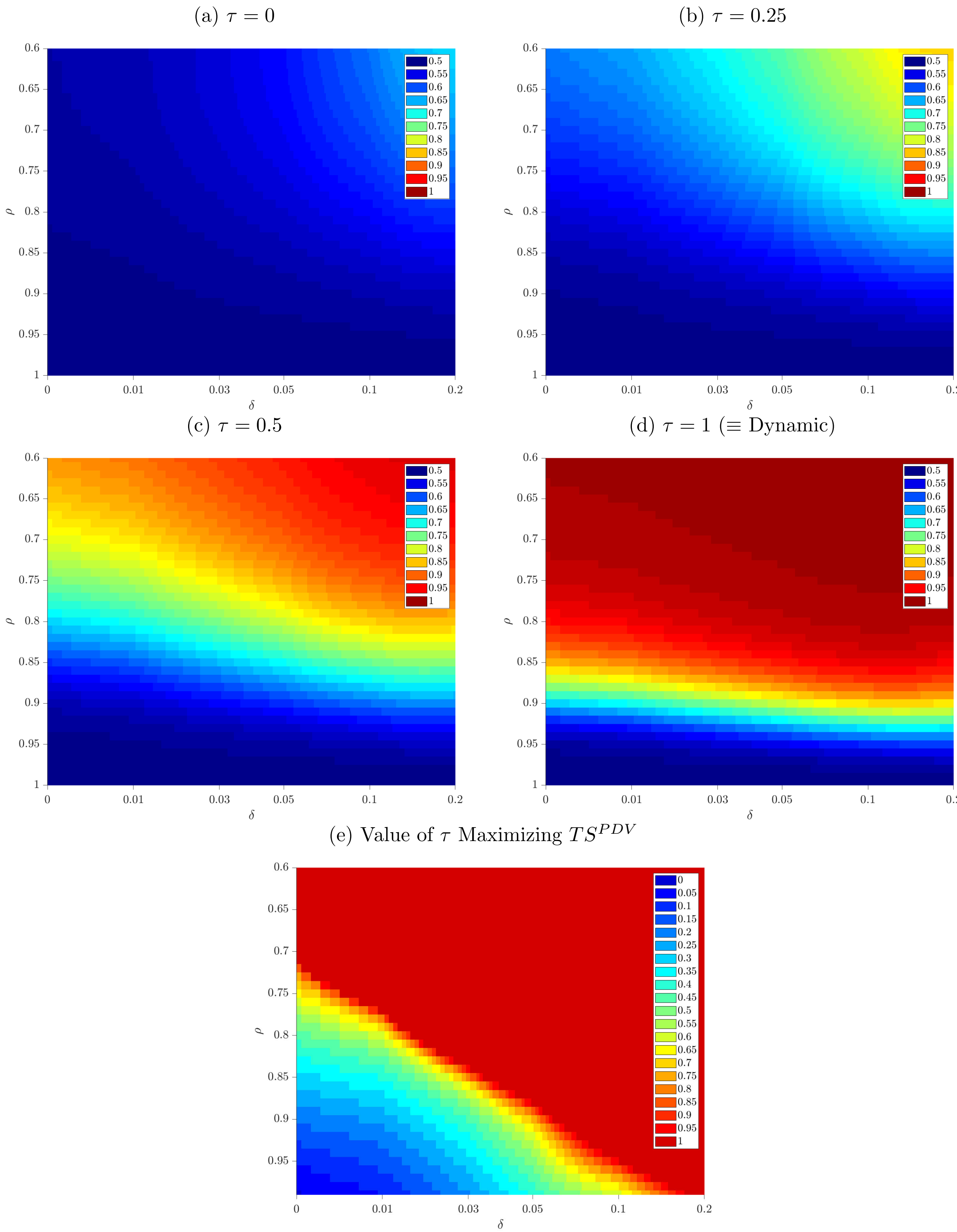
When depreciation is more likely, concentration is higher with dynamic seller behavior when $\tau = 0$, as long as LBD is not very limited. However, we still see that concentration increases relatively slowly with τ when sellers act statically.[35]

The effect of buyer bargaining power on efficiency. Figure 6(e) shows the level of τ which maximizes TS^{PDV} when sellers behave statically. Comparing this figure to Figure 5(b), we see a different pattern if LBD is important (i.e., $\rho < 0.75$) or depreciation is likely: with static seller behavior, efficiency is maximized by the buyer having all of the bargaining power, as it would be if costs were fixed, or almost all of the bargaining power, whereas with dynamic sellers, efficiency is typically maximized by bargaining power being equally split between buyers and sellers, or sellers having slightly more of the bargaining power, for almost all technologies. As we know that outcomes with static and dynamic sellers are identical if $\tau = 1$, these patterns imply that

[34] Of course, if the analyst compared other features of the equilibrium outcome, such as the size of margins over production costs, they would observe differences.

[35] Concentration is also monotonically increasing in τ, as it would if production costs were fixed (but asymmetric), whereas, with dynamic seller behavior, there can be a reduction in concentration as τ approaches 1 as dynamic incentives are eliminated. While $HHI(\mathbf{e})$ will increase with τ in every asymmetric state when sellers are static, expected HHI^t could fall when we average across states. Therefore, the monotonicity result is numerical.

Figure 6: Expected Value of the HHI^{32} in a Static Seller Equilibrium for Alternative Values of τ and the Value of τ that Maximizes TS^{PDV} Assuming Static Seller Behavior in the $M = 30$ Specification.



equilibrium outcomes when sellers are dynamic and buyers have moderate bargaining power are more efficient than equilibria with any level of buyer bargaining power and static sellers.

5 The Impact of Dynamics and Buyer Bargaining Power on Counterfactual Predictions

The goal of applied papers that use a model of dynamic competition is often to perform a counterfactual, such as designing a subsidy scheme (e.g., Irwin and Pavcnik (2004), Barwick, Kwon, Li, and Zahur (2025)) or predicting the effects of a proposed merger (e.g., An and Zhao (2019)). Here we consider stylized examples of both types of counterfactual. We will compare predictions with those from a dynamic model with seller price-setting and a static model that allows for bargaining. While it might seem unlikely that an economist would choose to use a static-seller assumption when they know there is LBD, we note that antitrust agencies commonly quantify price effects from mergers using calculations that assume static firm behavior, relying on qualitative arguments to suggest whether dynamics may exacerbate, or alleviate, anticompetitive concerns.[36]

5.1 Optimal Subsidies

Our first application considers a scheme to implement the social planner outcome as an equilibrium using state-specific transfers to, or from, a laggard that makes a sale.[37] We consider the structure of the scheme that optimally balances the benefits of rapid learning and the costs of single firm dominance, and the possible welfare costs of implementing a scheme whose design ignores buyer bargaining power.

Given social planner choice probabilities, $D_1^{SP}(\mathbf{e})$, it is simple to use our buyer choice probability formulation of the dynamic equilibrium to solve for M^2 equilibrium prices, and $\frac{M(M-1)}{2}$ transfers, $s_1(\mathbf{e})$ (positive for a subsidy, negative for a tax), using $\frac{M(M-1)}{2}$ equations

[36] For a discussion of the Federal Trade Commission's analysis of market definition and competitive effects when analyzing a merger with potential demand and supply dynamics, see Wosińska, Givens, Lau, Smith, Taylor, and Wallace (2021).

[37] With no outside good, sale probabilities in symmetric states are efficient so it is natural to restrict attention to transfers in asymmetric states.

$$p_1(\mathbf{e}) - p_2(\mathbf{e}) = \sigma \log \left(\frac{1}{D_1^{SP}(\mathbf{e})} - 1 \right), \tag{20}$$

and M^2 equations

$$\mathbf{p_1} + \mathbf{s_1} = \mathbf{\Phi}(\mathbf{D_1^{SP}}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1^{SP}} \circ \mathbf{\Phi}(\mathbf{D_1^{SP}})]. \tag{21}$$

The linearity of equation (21) in s_1 implies that the transfer scheme that can implement the social optimum will be unique, although this scheme could also support sub-optimal equilibria.

We calculate transfers for the $M = 30$ specification assuming that $\rho = 0.75$, implying $c(2) = 7.5$ and $c(e \geq m) = 3.25$, and $\delta = 0.023$, implying $\Delta(2) = 0.045$ and $\Delta(m) = 0.295$. For this technology, there is a unique equilibrium absent transfers for all τ. With either static or dynamic sellers, equilibrium HHI^{32} is less than the planner's solution would imply if $\tau = 0$.

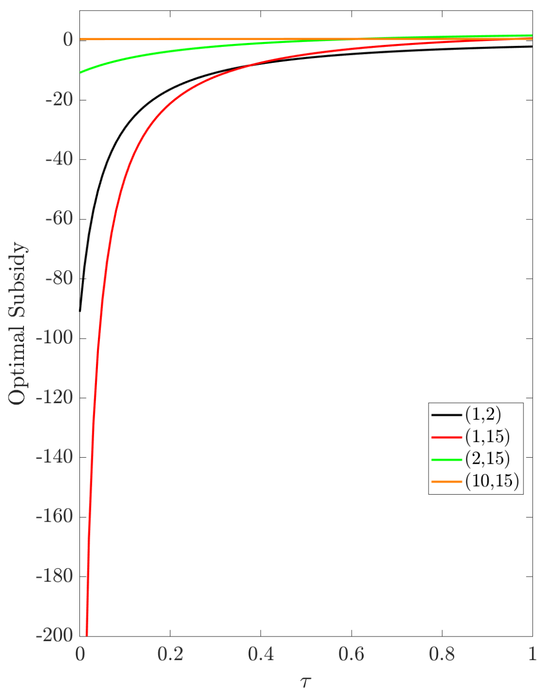
Figure 7(a) and (b) show the optimal transfers in several states as a function of τ, assuming either static or dynamic seller behavior. The optimal schemes are identical when $\tau = 1$. However, when $\tau = 0$, even the direction of the transfers can vary with the assumed seller behavior. Sales to the laggard are taxed if sellers are static, consistent with equilibrium concentration being too low.

In contrast, the need to subsidize laggard sales in states such as (1,2) and (2,15) when sellers are dynamic reflects the endogeneity of dynamic incentives. The logic can be seen by thinking through what happens in states (1,2) and (1,3). Under social planner control, the leader in state (1,3) would expect its lead to last 798 periods, compared to 88.5 periods in the no-transfer equilibrium. If the leader in state (1,2) expected its lead to last 798 periods if it wins and reaches (1,3), its dynamic incentive would be very large and it would win with higher probability than the planner would want. Therefore, a laggard subsidy in (1,2) is required to implement the planner outcome.

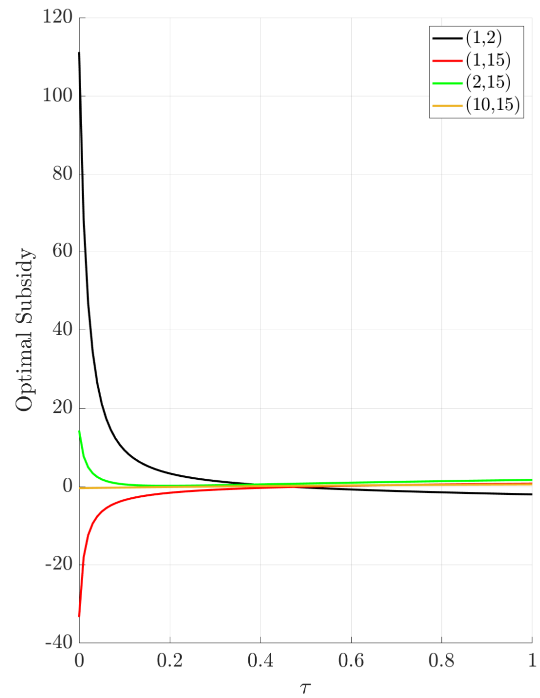
We also see that while the absolute magnitudes of transfers decrease with τ under both behavioral assumptions, the reduction is quicker when sellers are dynamic. When $\tau \approx 0.4$, dynamic seller transfers are small, consistent with the no-transfer equilibrium being close to achieving social planner efficiency.

Figure 7: Optimal Transfer Schemes and Welfare as Functions of τ in the $M = 30$ Specification if $\rho = 0.75$ and $\delta = 0.023$. Negative subsidies are taxes on the laggard when it makes a sale.

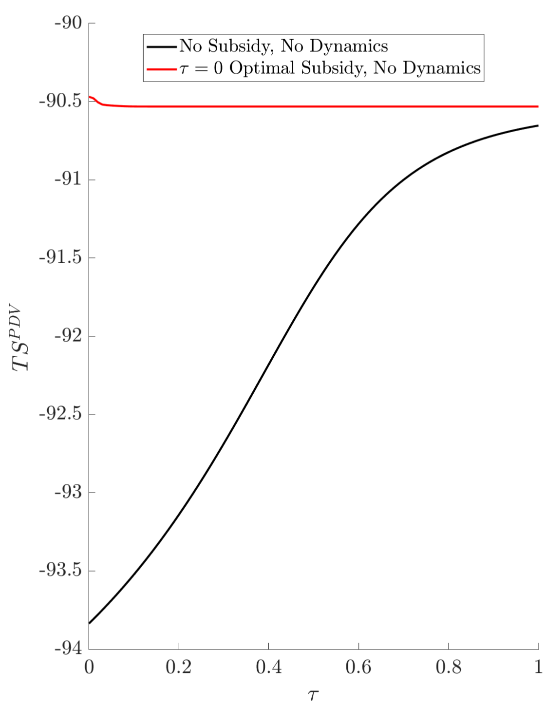
(a) Static $M = 30$: Optimal Subsidy Levels in Selected States



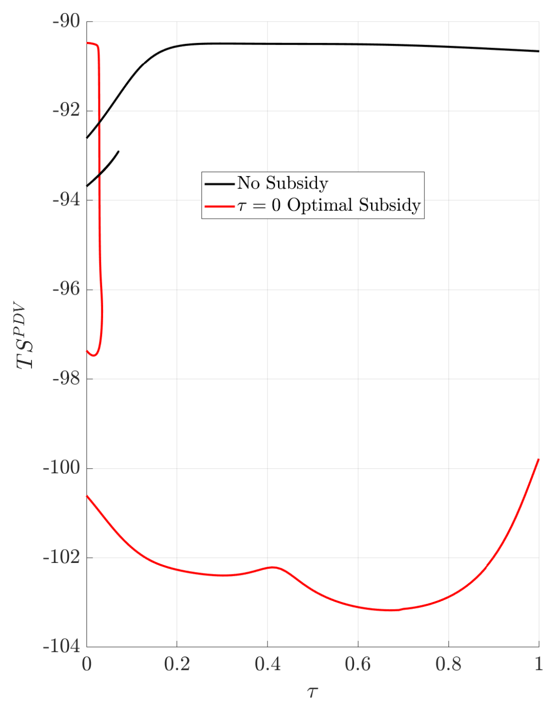
(b) Dynamic $M = 30$: Optimal Subsidy Levels in Selected States



(c) Static $M = 30$: TS^{PDV} with $\tau = 0$ Optimal Subsidies



(d) Dynamic $M = 30$: TS^{PDV} with $\tau = 0$ Optimal Subsidies



Panels (c) and (d) show the equilibrium TS^{PDV}, as functions of τ, with no tranfers (black line) and the tranfers that would be optimal if $\tau = 0$ (red line) given the relevant behavioral assumption.[38] This comparison is motivated by how the existing literature might lead an analyst to assume that $\tau = 0$ when making a policy recommendation.

If sellers are static, the imposition of $\tau = 0$ optimal transfers, some of which are large, creates only a small efficiency loss relative to the social optimum, and improves on the market equilibrium, for any level of τ. When sellers are dynamic, we know that the market equilibrium may be too concentrated when buyers have more than moderate bargaining power even without transfers. However, the pattern in panel (d) is still quite striking. The $\tau = 0$ optimal scheme supports multiple equilibria, and, unless the government can control equilibrium selection, the scheme might lower welfare even if $\tau = 0$. When $\tau \geq 0.04$, a threshold implying such a small level of buyer bargaining power that an analyst might assume it could be ignored, the $\tau = 0$ optimal scheme always lowers welfare, in this specification, compared to having no scheme at all. Appendix C.4 performs a similar analysis of the welfare impact of $\tau = 0$ subsidies when sellers are dynamic using the parameterization that we will introduce in the next sub-section to look at the effect of a horizontal merger. In these examples, we do not find multiple equilibria under the subsidy scheme, but, with two, three or four sellers, we continue to find that $\tau = 0$ subsidies lower welfare when buyers have more than moderate bargaining power.[39]

5.2 Horizontal Merger

Our second example involves the analysis of a horizontal merger. An and Zhao (2019) show that a model with dynamic sellers, LBD and learning spillovers across products within a firm predicts that the 1997 Boeing-McDonnell Douglas merger benefited customers.[40] Here we will assume that there are no synergies so that we can focus on how the value of τ affects the profitability and welfare

[38] We do not include the cost or the revenue generated from transfers in our calculations.

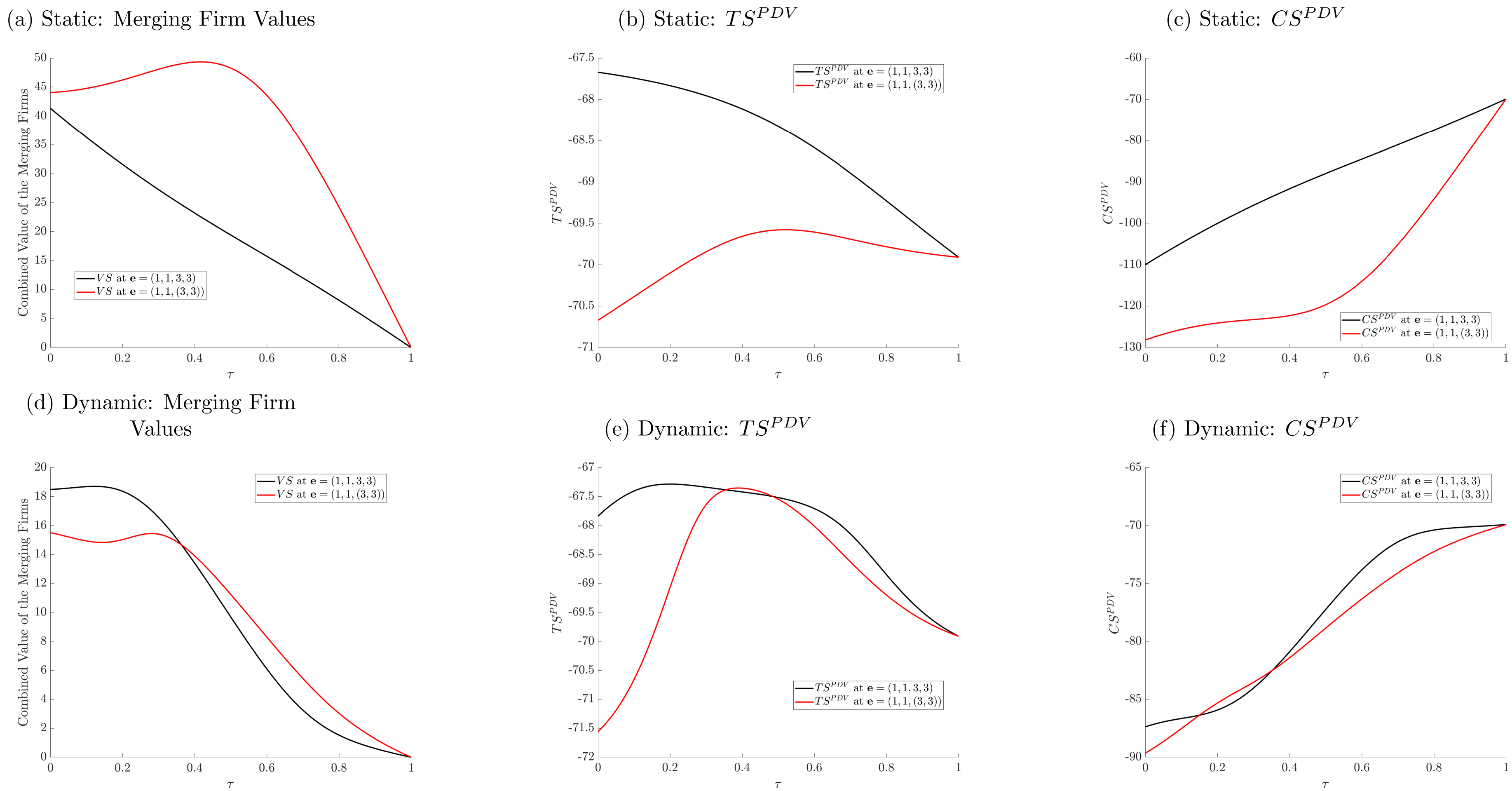
[39] The thresholds for welfare to be reduced by the subsidy scheme are $\tau \geq 0.1$, 0.16 and 0.18 with 2, 3 or 4 sellers respectively. Of course, these thresholds would be different if we used different parameterizations.

[40] Su, Yang, and Sweeting (2026) show that learning efficiencies, which lower cost and raise reliability, similarly rationalize the FTC's decision not to challenge the Boeing-Lockheed Martin United Launch Alliance joint venture.

impact of the proposed merger.[41]

[41] Gowrisankaran (1999) and Mermelstein, Nocke, Satterthwaite, and Whinston (2020) also consider mergers using dynamic models. None of these papers consider how buyer-seller bargaining could interact with dynamics to affect mergers, despite the attention that has recently been paid to incorporating bargaining into static models of mergers (Ho and Lee (2017), Gowrisankaran, Nevo, and Town (2015), Sheu and Taragin (2021)).

Figure 8: Effects of a Horizontal Merger Between Sellers with $e_3 = 3$ and $e_4 = 3$ Assuming Either Static Seller Behavior (panels (a)-(c)) or Dynamic Seller Behavior (panels (d)-(f)) as a Function of τ in a Model Where $M = 7$, $m = 5$, $\rho = 0.65$ and $\delta = 0.03$. Rivals' states when the merger is proposed are $e_1 = e_2 = 1$. The black lines indicate outcomes without the merger, and the red lines with the merger. When we calculate values or discounted surplus assuming static seller behavior we use the same discount factor, $\frac{1}{1.05}$, that we assume for the dynamic calculations.



To consider a meaningful merger, we extend the model to allow up to four firms and products, while reducing the number of know-how states ($m = 5$ and $M = 7$) to retain tractability. We set $\rho = 0.65$, so that $c(m) \approx 3.66$, and $\delta = 0.03$, so that $\Delta(m) \approx 0.14$. This depreciation assumption implies that all four products would likely stay at the bottom of their cost curves if they share sales equally. The remaining parameters, such as $\kappa = 10$ and $\sigma = 1$ are the same as in our $M = 30$ analysis.

Pre-merger there are four single product firms. We consider a one-time proposed merger between the know-how leaders in state (1,1,3,3), which is the merger that a standard analysis (e.g., based on market shares) is most likely to view as potentially anticompetitive.[42] We assume that the merger will generate no know-how efficiencies or spillovers in learning or forgetting across the merged firms' products, and that there are no fixed cost savings. Players understand that there will be no subsequent mergers. We assume the merged firm keeps both of its products and jointly negotiates their prices with buyers.[43] Appendix C.5 details the formulae characterizing the equilibrium of the model when firms have multiple products.

If production costs were fixed, standard Bertrand Nash results imply that, if $\tau < 1$, the merger would be profitable, and both buyer surplus and total surplus would decrease. We can simulate merger outcomes allowing costs to evolve but assuming static multiproduct seller pricing behavior. The black line in Figure 8(a) shows the sum of the values of the leaders in state (1,1,3,3), as a function of τ, if there is no merger. The red line shows the value of the merged firm as soon as the merger is consummated. Panels (b) and (c) show discounted total surplus and discounted consumer surplus.[44] If $\tau = 1$, the merger has no effect. However, for all $\tau < 1$, the merger is profitable, and it reduces efficiency and consumer surplus, i.e., the qualitative impacts of the merger are the same as if we ignored the fact that know-how could accumulate or depreciate.

[42] The combined pre-merger market shares of the leaders in state (1,1,3,3), with either static or dynamic seller behavior, would be above 94%, so the proposed merger might be seen as, essentially, a "merger to monopoly".

[43] This means that if the negotiation ends in disagreement the buyer will be unable to buy either product. Gowrisankaran, Nevo, and Town (2015) consider this assumption and an alternative assumption that the merged firm has to negotiate the price of each product separately.

[44] We calculate consumer surplus in a state as the expected value of ε of the chosen product less the expected price paid. As we exclude the vs, the discounted values of total and consumer surplus are negative.

Panels (d)-(f) show that qualitative conclusions can be different, and sensitive to the assumed level of buyer bargaining power, if we assume sellers behave dynamically. When $\tau = 0$, the merger is unprofitable, and it reduces efficiency and consumer surplus.[45] However, the merger is profitable for the merging parties if $\tau \geq 0.35$, it increases consumer surplus if $0.14 \leq \tau \leq 0.35$, and it is efficiency increasing if $0.35 \leq \tau \leq 0.5$.[46] Our example therefore illustrates how an analysis that allows for both dynamics and buyer bargaining power may be required to rationalize why a merger has been proposed and to evaluate whether it should be challenged.

Appendix C.6 compares the evolution of the industry when $\tau = 0$ and $\tau = 0.5$, assuming sellers are dynamic, to provide more insight into why profit and welfare outcomes vary with τ. It is only when buyers have sufficient bargaining power that the merged sellers can expect their leads to last long enough to be profitable. Buyer bargaining power also increases the probability that sales of low cost products produced by the merged firms will be made, raising efficiency.

Of course, our analysis has looked at one particular merger. Appendix C.6 also shows the effects on efficiency of mergers in the same state between either the two laggards or between a laggard and a leader. When sellers are static, these mergers decrease efficiency, although in the case of the laggard-laggard merger the effects are tiny, as, with or without the merger, the laggards are unlikely to make more than the occassional sale for a long time when they do not recognize their dynamic incentives. On the other hand, both mergers are efficiency-improving when τ is small if sellers are dynamic, whereas the leader-laggard merger is efficiency-reducing for moderate τ. Therefore the pattern that dynamics and bargaining power can interact in ways that affect the qualitative impact of a merger also holds for other merger types.

6 Conclusion

Models of dynamic competition are often motivated by references to industries, such as aircraft manufacturing, where buyers and sellers negotiate prices. This raises the question of whether the

[45]Of course, if we allowed for an appropriate fixed cost saving we could make any merger appear profitable.

[46]When $\tau = 0.35$ the merger is very close to neutral on all three measures, but there is no τ that makes the merger profitable, and both efficiency and buyer surplus increasing.

standard modeling assumption that buyers are price-takers could generate misleading predictions about market structure and efficiency, or policy choices. We have shown that, when sellers are dynamic, even moderate buyer bargaining power can lead to both market structure and economic distortions being quite different from what they would be if sellers set prices, and to different policy recommendations. Furthermore, we have shown that these changes reflect how bargaining and dynamics interact, so that both predictions and recommendations can be quite different from those derived from a model that accounts for buyer bargaining power but which ignores dynamics. As far as we are aware, our emphasis on the interaction between buyer bargaining power and dynamic competition, and our finding that buyer bargaining power often strengthens the dynamic incentives of a leader, relative to those of a laggard, are new, but they are potentially important in many applications.

Our analysis could be usefully extended in several directions. For example, while Deng, Jia, Leccese, and Sweeting (2025) provide some analysis of forward-looking buyers, bargaining power and exclusive contracts, realistic applications in many industries would need to account for how buyers purchase multiple units from different manufacturers over time, while negotiating volume discounts with several of them. As many of our results reflect how the buyer bargaining weight parameter affects continuation values, as well as the split of current surplus, they also highlight the need for econometric methods that can, in these situations, estimate the distribution of bargaining power consistently.

References

Aguirregabiria, V., A. Collard-Wexler, and S. P. Ryan (2021): "Dynamic Games in Empirical Industrial Organization," in *Handbook of Industrial Organization*, vol. 4, pp. 225–343. Elsevier.

An, Y., and W. Zhao (2019): "Dynamic Efficiencies of the 1997 Boeing-McDonnell Douglas Merger," *RAND Journal of Economics*, 50(3), 666–694.

Asker, J., C. Fershtman, J. Jeon, and A. Pakes (2020): "A Computational Framework for Analyzing Dynamic Auctions: The Market Impact of Information Sharing," *RAND Journal of Economics*, 51(3), 805–839.

Athey, S., and A. Schmutzler (2001): "Investment and Market Dominance," *RAND Journal of Economics*, 32(1), 1–26.

Barwick, P. J., H.-S. Kwon, S. Li, and N. B. Zahur (2025): "Drive Down The Cost: Learning-by-Doing and Government Policies in the Global EV Battery Industry," Discussion paper, National Bureau of Economic Research.

Bedre-Defolie, Ö. (2012): "Vertical Coordination Through Renegotiation," *International Journal of Industrial Organization*, 30(6), 553–563.

Benkard, C. L. (2000): "Learning and Forgetting: The Dynamics of Aircraft Production," *American Economic Review*, 90(4), 1034–1054.

——— (2004): "A Dynamic Analysis of the Market for Wide-Bodied Commercial Aircraft," *Review of Economic Studies*, 71(3), 581–611.

Besanko, D., U. Doraszelski, and Y. Kryukov (2014): "The Economics of Predation: What Drives Pricing When There is Learning-by-Doing?," *American Economic Review*, 104(3), 868–97.

——— (2019): "How Efficient is Dynamic Competition? The Case of Price as Investment," *American Economic Review*, 109(9), 3339–64.

Besanko, D., U. Doraszelski, Y. Kryukov, and M. Satterthwaite (2010): "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics," *Econometrica*, 78(2), 453–508.

Biglaiser, G., and J. Crémer (2020): "The Value of Incumbency When Platforms Face Heterogeneous Customers," *American Economic Journal: Microeconomics*, 12(4), 229–269.

Biglaiser, G., J. Crémer, and A. Mantovani (2024): "The Economics of the Cloud," Discussion paper, University of North Carolina.

Budd, C., C. Harris, and J. Vickers (1993): "A Model of the Evolution of Duopoly: Does the Asymmetry Between Firms Tend to Increase or Decrease?," *Review of Economic Studies*, 60(3), 543–573.

Cabral, L. (2011): "Dynamic Price Competition with Network Effects," *Review of Economic Studies*, 78(1), 83–111.

Cabral, L. M., and M. H. Riordan (1994): "The Learning Curve, Market Dominance, and Predatory Pricing," *Econometrica*, 62(5), 1115–1140.

Collard-Wexler, A., G. Gowrisankaran, and R. S. Lee (2019): "Nash-in-Nash Bargaining: A Microfoundation for Applied Work," *Journal of Political Economy*, 127(1), 163–195.

Covert, T., and R. Sweeney (2022): "Winds of Change: Estimating Learning by Doing Without Cost or Input Data," Discussion paper, Boston College.

Crawford, G. S., R. S. Lee, M. D. Whinston, and A. Yurukoglu (2018): "The Welfare Effects of Vertical Integration in Multichannel Television markets," *Econometrica*, 86(3), 891–954.

Dasgupta, P., and J. Stiglitz (1988): "Learning-By-Doing, Market Structure and Industrial and Trade Policies," *Oxford Economic Papers*, 40(2), 246–268.

Deng, S., D. Jia, M. Leccese, and A. Sweeting (2025): "Dynamic Competition with Bargaining: Implications for Subsidy and Competition Policies," Discussion paper, University of Maryland.

Dequiedt, V., and D. Martimort (2015): "Vertical Contracting with Informational Opportunism," *American Economic Review*, 105(7), 2141–2182.

Do, J., and J. Miklós-Thal (2025): "Opportunism in Vertical Contracting: A Dynamic Perspective," *RAND Journal of Economics*, 56(4), 708–737.

Doraszelski, U., and M. Satterthwaite (2010): "Computable Markov-Perfect Industry Dynamics," *RAND Journal of Economics*, 41(2), 215–243.

Dorn, J. (2025): "Dynamic Bargaining Between Hospitals and Insurers," Discussion paper, Cornell University.

Ericson, R., and A. Pakes (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies*, 62(1), 53–82.

Farrell, J., and P. Klemperer (2007): "Coordination and Lock-In: Competition with Switching Costs and Network Effects," in *Handbook of Industrial Organization*, ed. by M. Armstrong, and R. Porter, vol. 3, pp. 1967–2072. Elsevier.

Flamini, F. (2012): "Recursive Bargaining with Dynamic Accumulation," in *Distributed Decision Making and Control*, ed. by R. Johansson, and A. Rantzer, pp. 131–144. Springer.

Fudenberg, D., and J. Tirole (1983): "Learning-by-Doing and Market Performance," *Bell Journal of Economics*, 14(2), 522–530.

Ghemawat, P. (1985): "Building Strategy on the Experience Curve," *Harvard Business Review*, 63(2), 143–149.

Gowrisankaran, G. (1999): "A Dynamic Model of Endogenous Horizontal Mergers," *RAND Journal of Economics*, 30(1), 56–83.

Gowrisankaran, G., A. Nevo, and R. Town (2015): "Mergers When Prices Are Negotiated: Evidence from the Hospital Industry," *American Economic Review*, 105(1), 172–203.

Grennan, M. (2013): "Price Discrimination and Bargaining: Empirical Evidence from Medical Devices," *American Economic Review*, 103(1), 145–177.

Halaburda, H., B. Jullien, and Y. Yehezkel (2020): "Dynamic Competition with Network Externalities: How History Matters," *RAND Journal of Economics*, 51(1), 3–31.

Ho, K., and R. S. Lee (2017): "Insurer Competition in Health Care Markets," *Econometrica*, 85(2), 379–417.

Ho, K., and R. S. Lee (2019): "Equilibrium Provider Networks: Bargaining and Exclusion in Health Care Markets," *American Economic Review*, 109(2), 473–522.

Horn, H., and A. Wolinsky (1988): "Bilateral Monopolies and Incentives for Merger," *RAND Journal of Economics*, 19(3), 408–419.

Houba, H., K. Sneek, and F. Vardy (2000): "Can Negotiations Prevent Fish Wars?," *Journal of Economic Dynamics and Control*, 24(8), 1265–1280.

Irwin, D. A., and N. Pavcnik (2004): "Airbus Versus Boeing Revisited: International Competition in the Aircraft Market," *Journal of International conomics*, 64(2), 223–245.

Iskhakov, F., J. Rust, and B. Schjerning (2016): "Recursive Lexicographical Search: Finding All Markov Perfect Equilibria of Finite State Directional Dynamic Games," *Review of Economic Studies*, 83(2), 658–703.

Kalai, E. (1977): "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica*, 45(7), 1623–1630.

Kim, M. (2014): "Strategic Responses to Used Goods Markets: Airbus and Boeing," Discussion paper, University of Oklahoma.

Leahy, D., and J. P. Neary (1999): "Learning by Doing, Precommitment and Infant-Industry Promotion," *Review of Economic Studies*, 66(2), 447–474.

Lee, R. S., and K. Fong (2013): "Markov Perfect Network Formation: An Applied Framework for Bilateral Oligopoly and Bargaining in Buyer-Seller Networks," Discussion paper, New York University.

Maskin, E., and J. Tirole (2001): "Markov Perfect Equilibrium: I. Observable Actions," *Journal of Economic Theory*, 100(2), 191–219.

McAfee, R. P., and M. Schwartz (1994): "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity," *American Economic Review*, 84(1), 210–230.

Mermelstein, B., V. Nocke, M. A. Satterthwaite, and M. D. Whinston (2020): "Internal versus External Growth in Industries with Scale Economies: A Computational Model of Optimal Merger Policy," *Journal of Political Economy*, 128(1), 301–341.

Pakes, A., and P. McGuire (1994): "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model," *RAND Journal of Economics*, 25(4), 555–589.

Sheu, G., and C. Taragin (2021): "Simulating Mergers in a Vertical Supply Chain with Bargaining," *RAND Journal of Economics*, 52(3), 596–632.

Siebert, R. B. (2010): "Learning-by-Doing and Cannibalization Effects at Multi-Vintage Firms: Evidence from the Semiconductor Industry," *B.E. Journal of Economic Analysis & Policy*, 10(1).

Sorger, G. (2006): "Recursive Nash Bargaining over a Productive Asset," *Journal of Economic Dynamics and Control*, 30(12), 2637–2659.

Spence, A. M. (1981): "The Learning Curve and Competition," *Bell Journal of Economics*, 12(1), 49–70.

Su, R., C. Yang, and A. Sweeting (2026): "Competition, Procurement and Learning-by-Doing in the Space Launch Industry," Discussion paper, National Bureau of Economic Research.

Sweeting, A., D. Jia, S. Hui, and X. Yao (2022): "Dynamic Price Competition, Learning-by-Doing, and Strategic Buyers," *American Economic Review*, 112(4), 1311–33.

Sweeting, A., J. W. Roberts, and C. Gedge (2020): "A Model of Dynamic Limit Pricing with an Application to the Airline Industry," *Journal of Political Economy*, 128(3), 1148–1193.

Sweeting, A., X. Tao, and X. Yao (2024): "Dynamic Games with Asymmetric Information: Implications for Mergers," *American Economic Journal: Microeconomics*, 16(3), 345–373.

Wosińska, M., D. Givens, Y. Lau, D. S. Smith, C. Taylor, and B. Wallace (2021): "Economics at the FTC: Multi-Level Marketing and a Coal Joint Venture," *Review of Industrial Organization*, 59(4), 629–650.

Yang, C. (2020): "Vertical Structure and Innovation: A Study of the SoC and Smartphone Industries," *RAND Journal of Economics*, 51(3), 739–785.

Online Appendices For "Dynamic Seller Competition When Buyers Have Bargaining Power" by Deng, Jia, Leccese and Sweeting

A Numerical Methods for Finding Equilibria

A.1 Homotopies

This Appendix details our implementation of the homotopy algorithm for a model where $M = 30$. We focus on how we use a sequence of homotopies to try to enumerate the number of equilibria that exist for different values of (ρ, δ) for given values of τ. Our implementation of other homotopies, for example, by varying τ, is similar to a single step in this sequence. We describe the procedure when we use the price and value formulation of the equilibrium equations. Identical procedures can be applied to the choice probability formulation, except that we calculate the prices and values implied by the choice probabilities in order to check whether the identified solutions are different enough to be labeled as distinct equilibria.

A.1.1 Preliminaries

We identify equilibria at particular gridpoints in (ρ, δ) space. We specify a 201-point evenly-spaced grid for the forgetting rate $\delta \in [0, 0.2]$ and a 41-point evenly-spaced grid for the learning progress ratio $\rho \in [0.6, 1]$. The state space of the game is defined by a (30×30) grid of values of the know-how of each firm.

A.1.2 System of Equations Defining Equilibrium

An MPNE is defined by a system of equations, i.e., one V_1^{S*} equation (text equation (2)) for each of 900 states and one p_1^* equation (text equation (6)), for each of 900 states. The grouping of all of these equations is denoted F.

A.1.3 Homotopy Algorithm: Overview

The idea of a homotopy is to trace out an equilibrium correspondence as one of the parameters of interest is changed, holding the others fixed. Starting from any equilibrium, the numerical algorithm traces a path where a parameter (such as δ), and the vectors $V^{S*}(\mathbf{e})$ and $p^*(\mathbf{e})$ are changed together so that the equations F continue to hold, by solving a system of differential equations. The differential equation solver does not return equilibria exactly at the gridpoints so it is necessary to interpolate between the solutions returned by the solver. Homotopies can be run starting from different equilibria, varying different parameters and switching the direction in which the parameter changes. When these different homotopies return interpolated solutions at the same gridpoint it is necessary to define a numerical rule for when two different solutions should be counted as different equilibria.

A.1.4 Procedure Details

Step 1: Finding Equilibria for $\delta = 0$. The first step is to find an equilibrium (i.e., a solution to the 1,800 equations) for $\delta = 0$ for each value of ρ on the grid. There will be a unique MPNE for $\delta = 0$, as, in this case, movements through the state space are unidirectional, so that the state will eventually end up in the state (M, M) where no more learning is possible.

We solve for an equilibrium using the Levenberg-Marquardt algorithm implemented using `fsolve` in MATLAB, where we supply analytic gradients for each equation. The solution for the previous value of ρ is used as a starting value. To ensure that the solutions are precise, we use a tolerance of 10^{-7} for the sum of squared values of each equation, and a relative tolerance of 10^{-14} for the price and value variables that we are solving for.

Step 2: δ-Homotopies. Using the notation of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS), we explore the correspondence

$$F^{-1}(\rho) = \{(\mathbf{V}^*, \mathbf{p}^*, \delta) | F(\mathbf{V}^*, \mathbf{p}^*; \rho, \delta) = \mathbf{0}, \quad \delta \in [0, 0.2]\}, \tag{22}$$

The homotopy approach follows the correspondence as a parameter, labeled s, changes. Here we are thinking of changing δ, and fixing ρ. Denoting $\mathbf{x} = (\mathbf{V}^*, \mathbf{p}^*)$, $F(\mathbf{x}(s), \delta(s), \rho) = \mathbf{0}$ can be implicitly differentiated to find how $\mathbf{x}$ and δ must change for the equations to continue to hold as s changes.

$$\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}} \mathbf{x}'(s) + \frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta} \delta'(s) = \mathbf{0} \tag{23}$$

where $\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}}$ is a (1,800 x 1,800) matrix, $\mathbf{x}'(s)$ and $\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta}$ are both (1,800 x 1) vectors and $\delta'(s)$ is a scalar. The solution to these differential equations will have the following form, where $y_i'(s)$ is the derivative of the ith element of $\mathbf{y}(s) = (\mathbf{x}(s), \delta(s))$,

$$y_i'(s) = (-1)^{i+1} \det \left(\left(\frac{\partial F(\mathbf{y}(s), \rho)}{\partial \mathbf{y}} \right)_{-i} \right) \tag{24}$$

where $_{-i}$ means that the ith column is removed from the (1,801 x 1,801) matrix $\frac{\partial F(\mathbf{y}(s), \rho)}{\partial \mathbf{y}}$.

To implement the path-following procedure, we use the FORTRAN routine FIXPNS from HOMPACK90, with the ADIFOR 2.0D automatic differentiation package used to evaluate the sparse Jacobian $\frac{\partial F(\mathbf{y}(s), \rho)}{\partial \mathbf{y}}$ and the STEPNS routine is used to find the next point on the path.[47,48]

The FIXPNS routine will return solutions at values of δ that are not equal to the gridpoints. Therefore we adjust the code so that after *each* step, the algorithm checks whether a gridpoint has been passed and, if so, the routine ROOTNX is used to calculate the equilibrium at the gridpoint, using information on the solutions at either side.[49]

The time taken to run a homotopy is usually between one hour and seven hours, when it is run on UMD's BSWIFT cluster (a moderately sized cluster for the School of Behavioral and Social Sciences).

[47] STEPNS is a predictor-corrector algorithm where Hermite cubic interpolation is used to guess the next point, and an iterative procedure is then used to return to the path.

[48] For details of the HOMPACK subroutines, please consult the manual of the algorithm available at `https://users.wpi.edu/~walker/Papers/hompack90,ACM-TOMS_23,1997,514-549.pdf`.

[49] It can happen that the ROOTNX routine stops prematurely so that the returned solution is not exactly at the gridpoint value of δ. We do not use the small proportion of solutions where the difference is more than 10^{-6}. Varying this threshold does not affect the reported results. We also need to decide whether the equations have been solved accurately enough so that the values and strategies can be treated as equilibria. We use solutions where the value of each equation residual is less than 10^{-10}. Otherwise, the solution is rejected. In practice, the rejected solutions typically have residuals that are much larger than 10^{-10}.

Step 3: Enumerating Equilibria. Once we have collected the solutions at each of the (ρ, δ) gridpoints we need to identify which solutions represent distinct equilibria, taking into account that small differences may arise because of numerical differences that are within our tolerances. For this paper, we use the rule that solutions count as different equilibria if at least one element of the price vector differs by more than 0.001.

Step 4: ρ-Homotopies. With a set of equilibria from the δ-homotopies in hand, we can perform the next round of our criss-crossing procedure which alternates ρ-homotopies and δ-homotopies, which we run in both directions (e.g., decreasing ρ as well as increasing ρ). We use equilibria found in the last round as starting points.[50]

This second round of homotopies can also help us to deal with gridpoints where the first round δ-homotopies identify no equilibria because a homotopy run stops or stalls (taking a long sequence of infinitesimally small steps). As noted by BDKS p. 467, the homotopies may stop if they reach a point where the evaluated Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ has less than full rank. Suppose, for example, that the δ-homotopy for $\rho = 0.8$ stops at $\delta = 0.1$, so we have no equilibria for δ values above 0.1. Homotopies that are run from gridpoints where we did find equilibria with higher values of δ and higher or lower values of ρ may fill in some of the missing equilibria.

Step 5: Repeat Steps 3, 2 and 4 to Identify Additional Equilibria Using New Equilibria as Starting Points. We use the procedures described in Step 3 to identify new equilibria at the gridpoints. These new equilibria are used to start new sets of δ-homotopies, which in turn can identify equilibria that can be used for new sets of ρ-homotopies. This iterative process is continued until the number of additional equilibria that are identified in a round has no noticeable effect on the heatmaps which show the number of equilibria. When $\tau = 0$, this happens after 8 rounds.

[50]In practice, using all new equilibria could be computationally prohibitive. We therefore use an algorithm that continues to add new groups of 10,000 starting points when we find that using additional starting points yields a significant number of equilibria that have not been identified before. We have experimented with different rules, and have found that no alternatives find noticeably more equilibria than the algorithm that we use.

B Analytical Proofs

B.1 Proof of Lemma 1

Lemma 1 *Consider a state* (e_1, e_2) *and hold fixed seller opportunity costs,* $\widehat{c}_1$ *and* $\widehat{c}_2$*. For all* τ*, equilibrium prices are unique.*

Proof. Fix a state and fixed opportunity costs $\widehat{c}_1$ and $\widehat{c}_2$. Combining the logit inversion equation (8) with the markup equation (9), an equilibrium choice probability D_1^* must solve

$$\sigma \log \frac{1 - D_1^*}{D_1^*} = \widehat{c}_1 - \widehat{c}_2 + \Phi(D_1^*, \tau) - \Phi(1 - D_1^*, \tau), \tag{25}$$

where

$$\Phi(D, \tau) = \frac{(1-\tau)\sigma \log \frac{1}{1-D}}{\tau D + (1-\tau)(1-D)\log \frac{1}{1-D}}. \tag{26}$$

The left-hand side of equation (25) is strictly decreasing in D_1^*. If $\tau < 1$, $\Phi(D, \tau)$ can be written as

$$\Phi(D, \tau) = \frac{\sigma}{\frac{\tau}{1-\tau}\frac{D}{\log \frac{1}{1-D}} + 1 - D}, \tag{27}$$

and the denominator is strictly decreasing in D, so $\Phi(D, \tau)$ is strictly increasing in D. Thus the right-hand side of equation (25) is strictly increasing in D_1^*. If $\tau = 1$, $\Phi(D, 1) = 0$, so that the right-hand side of (25) is constant. Either way, since the left-hand side diverges to $+\infty$ as $D_1^* \downarrow 0$ and to $-\infty$ as $D_1^* \uparrow 1$, there is a unique solution. Prices are therefore uniquely determined by $p_i = \widehat{c}_i + \Phi(D_i, \tau)$. ■

B.2 Proof of Proposition 1

Proposition 1 *In the dynamic model there will be a unique symmetric MPNE if*

1. $\delta = 0$, *or*

2. $\tau = 1$, *or*

3. $M = 2$.

Proof. Part 1. If $\delta = 0$, know-how never depreciates. Once play leaves a state, it cannot return, and the process eventually reaches (M, M). At state (M, M), the state remains (M, M) regardless of which seller makes a sale, so dynamic incentives are zero, opportunity costs equal production costs, and Lemma 1 implies a unique continuation equilibrium. Applying the same backward induction argument as in Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS), Proposition 3, and applying Lemma 1 at each state, gives uniqueness of the symmetric MPNE for every τ.

Part 2. If $\tau = 1$, then $\Phi(D, 1) = 0$ in every state. Hence the seller profit vector is zero, and equation (11) gives $\mathbf{V}_i^S = \mathbf{0}$ for both sellers. Opportunity costs equal production costs, prices equal production costs, and the logit probabilities are uniquely pinned down by current costs. This gives the unique symmetric MPNE.

Part 3. For all $M = 2$ proofs, write

$$\Delta_1 := \Delta(1), \qquad \Delta_2 := \Delta(2), \qquad \Delta c := c(1) - c(2) > 0,$$

and let

$$x(\tau) := D_1^*((1, 2); \tau)$$

denote the laggard's sale probability in the asymmetric state. When using vectors, we order the states

$$(1, 1), (1, 2), (2, 1), (2, 2).$$

Uniqueness when $\tau = 1$ follows from part 2, so we only consider $\tau < 1$. In a symmetric equilibrium,

$$D_1^* = \left(\frac{1}{2}, x, 1 - x, \frac{1}{2}\right), \qquad D_2^* = \left(\frac{1}{2}, 1 - x, x, \frac{1}{2}\right).$$

Therefore, it suffices to show that the equilibrium condition in state (1,2) uniquely pins down x.

The equilibrium condition in state $(1, 2)$ follows from the same logit-inversion and markup-over-opportunity-cost logic used to derive equation (25):

$$\Delta c = \sigma \log \frac{1-x}{x} - \Phi(x,\tau) + \Phi(1-x,\tau) + DI_1 - DI_2.$$

where $DI_i = \mu^S_{i,i}(1,2) - \mu^S_{i,j}(1,2)$, $j \neq i$, is seller i's dynamic incentive in state $(1,2)$. Let

$$d := \big(\Delta_1\Delta_2,\ \Delta_1(1-\Delta_2) - 1,\ (1-\Delta_1)\Delta_2,\ (1-\Delta_1)(1-\Delta_2)\big),$$

then $DI_1 - DI_2 = \beta d \cdot (V^S_1 + V^S_2)$. Therefore the equilibrium condition in state $(1,2)$ can be written as

$$\Delta c = \sigma \log \frac{1-x}{x} - \Phi(x,\tau) + \Phi(1-x,\tau) + \beta d \cdot (V^S_1 + V^S_2). \tag{28}$$

Let $\Pi(D,\tau) := D\Phi(D,\tau)$. By equation (11), seller 1's value vector can be written as

$$V^S_1 = (I - \beta Q_2)^{-1} \begin{pmatrix} \Pi(1/2,\tau) \\ \Pi(x,\tau) \\ \Pi(1-x,\tau) \\ \Pi(1/2,\tau) \end{pmatrix}.$$

This means that the continuation term in equation (28) is linear in the three profit terms:

$$d \cdot (V^S_1 + V^S_2) = \eta_{1/2}\Pi(1/2,\tau) + \eta_x \Pi(x,\tau) + \eta_{1-x}\Pi(1-x,\tau), \tag{29}$$

with

$$\eta_{1-x} = -\frac{\Delta_1\Delta_2 + (1-\Delta_1)(1-\Delta_2)}{1-\beta\Delta_1(1-\Delta_2)} \in [-1,0], \tag{30}$$

$$\eta_x = \frac{P(\beta)}{(\beta\Delta_1 - 1)\,[1-\beta(1-\Delta_2)]\,[1-\beta\Delta_1(1-\Delta_2)]} \leq 0, \tag{31}$$

where

$$P(\beta)=1-\Delta_1-\Delta_2+2\Delta_1\Delta_2+2\beta\Delta_1(1-\Delta_2)(\Delta_1-1-\Delta_2)+\beta^2(1-\Delta_2)\Delta_1[\Delta_1(1-\Delta_1)+\Delta_2(1-\Delta_2)]. \tag{32}$$

The remaining coefficient is pinned down by

$$\eta_{1/2}=-(\eta_x+\eta_{1-x}), \tag{33}$$

because the components of d sum to zero. The sign of η_x follows because the denominator in equation (31) is negative and the quadratic P is nonnegative: its leading coefficient and constant are nonnegative, and its discriminant is

$$-4\Delta_1\Delta_2(1-\Delta_1)(1-\Delta_2)(\Delta_1+\Delta_2-1)^2\leq 0. \tag{34}$$

Let $\Psi(x,\tau)$ denote the right-hand side of equation (28), after substituting equation (29) into it. That is,

$$\Psi(x,\tau):=\sigma\log\frac{1-x}{x}-\Phi(x,\tau)+\Phi(1-x,\tau)+\beta\left[\eta_{1/2}\Pi(1/2,\tau)+\eta_x\Pi(x,\tau)+\eta_{1-x}\Pi(1-x,\tau)\right].$$

We now show that $\Psi(\cdot,\tau)$ is strictly decreasing on $(0,1/2)$. Differentiating equation (28) and using equation (29),

$$\Psi_x=-\sigma\left(\frac{1}{x}+\frac{1}{1-x}\right)-\Phi_D(x,\tau)-\Phi_D(1-x,\tau)+\beta\left[\eta_x\Pi_D(x,\tau)+\eta_{1-x}\frac{d}{dx}\Pi(1-x,\tau)\right].$$

Since $\Pi_D(D,\tau)=\Phi(D,\tau)+D\Phi_D(D,\tau)>0$, $\frac{d}{dx}\Pi(1-x,\tau)<0$, $\eta_x\leq 0$, and $\eta_{1-x}\in[-1,0]$,

$$\Psi_x\leq-\sigma\left(\frac{1}{x}+\frac{1}{1-x}\right)-\Phi_D(x,\tau)-\Phi_D(1-x,\tau)+\beta\left[\Phi(1-x,\tau)+(1-x)\Phi_D(1-x,\tau)\right].$$

Using $\Phi(D,\tau) \le \Phi(D,0) = \sigma/(1-D)$ gives

$$\Psi_x \le -\sigma\frac{1-\beta}{x} - \sigma\frac{1}{1-x} - \Phi_D(x,\tau) - [1-\beta(1-x)]\Phi_D(1-x,\tau) < 0.$$

Also, $\Psi(x,\tau) \to +\infty$ as $x \downarrow 0$, and $\Psi(1/2,\tau) = 0$ because both sellers then have the same current-profit vector and the components of d sum to zero. Since $\Delta c > 0$, equation (28) has exactly one solution in $(0, 1/2)$. This solution pins down the unique symmetric equilibrium. ■

B.3 Proof of Proposition 2

It is useful to establish the following lemma first.

Lemma B.1 *For $D \in (0,1)$ and $\tau \in [0,1]$, define*

$$M(D,\tau) := -\Phi_\tau(D,\tau) = \frac{\sigma D \log\frac{1}{1-D}}{\left[\tau D + (1-\tau)(1-D)\log\frac{1}{1-D}\right]^2}. \tag{35}$$

For a fixed τ, $M(D,\tau)$ is strictly increasing in D.

Proof. Write $L(D) := \log\frac{1}{1-D}$. When $\tau < 1$, $M(D,\tau) = m(D,z)/(1-\tau)^2$, where

$$m(D,z) := \frac{DL(D)}{[zD + (1-D)L(D)]^2}, \qquad z := \frac{\tau}{1-\tau} \ge 0.$$

We have that

$$m_D(D,z) = \frac{zD\left[\frac{D}{1-D} - L(D)\right] + L(D)\left[(1+D)L(D) - D\right]}{[zD + (1-D)L(D)]^3}.$$

Both bracketed terms are strictly positive because

$$\frac{D}{1-D} > L(D) > \frac{D}{1+D} \qquad \text{for all } D \in (0,1).$$

Hence $m_D(D,z) > 0$, and therefore $M_D(D,\tau) > 0$ for $\tau < 1$. When $\tau = 1$, $M(D,1) = \sigma L(D)/D$, and

$$\frac{d}{dD}\frac{L(D)}{D} = \frac{\frac{D}{1-D} - L(D)}{D^2} > 0.$$

This proves the claim for $\tau \in [0, 1]$. ■

Proposition 2 *Consider a one-shot bargaining game where sellers 1 and 2 have fixed opportunity costs of sale, $c_1 - DI_1$ and $c_2 - DI_2$ respectively. The following three properties hold for all technology and taste parameters:*

1. *if and only if $c_i - DI_i < c_j - DI_j$, then $D_i^* > \frac{1}{2}$ for all τ.*

2. *the static markups, $\Phi(D_k^*(\mathbf{e}), \tau)$, and therefore the prices, of both firms strictly decrease in τ.*

3. *if $c_i - DI_i < c_j - DI_j$, then, for any change in τ, the static markup of firm i decreases more than the markup of firm j, and D_i^* is strictly increasing in τ.*

Proof. Write each seller's fixed opportunity cost as

$$\widehat{c}_k := c_k - DI_k, \qquad k = i, j.$$

By Proposition 1, for every τ, there is a unique equilibrium choice probability. Seller i's equilibrium share solves

$$\sigma \log \frac{1 - D_i^*}{D_i^*} = \widehat{c}_i - \widehat{c}_j + \Phi(D_i^*, \tau) - \Phi(1 - D_i^*, \tau). \tag{36}$$

<u>*Part 1.*</u> If $\widehat{c}_i = \widehat{c}_j$, the unique solution is symmetric. If $\widehat{c}_i < \widehat{c}_j$, then evaluating equation (36) at $D_i = 1/2$ gives a positive left-minus-right gap, while this gap is strictly decreasing in D_i. Hence the unique solution has $D_i^* > 1/2$. The converse follows by reversing the sellers' identities.

<u>*Part 2.*</u> If $\widehat{c}_i = \widehat{c}_j$, then $D_i^* = D_j^* = 1/2$ for all τ, and $\Phi(1/2, \tau)$ is strictly decreasing in τ. Suppose instead that $\widehat{c}_i < \widehat{c}_j$, and set

$$D := D_i^* > 1/2.$$

Define

$$F(D,\tau) := \sigma \log \frac{1-D}{D} - \widehat{c}_i + \widehat{c}_j - \Phi(D,\tau) + \Phi(1-D,\tau).$$

The equilibrium condition is $F(D,\tau)=0$. The proof of Proposition 1 gives $F_D<0$. Using Lemma B.1,

$$F_\tau = M(D,\tau) - M(1-D,\tau) > 0,$$

because $D > 1/2$. Hence the implicit function theorem gives

$$D_\tau = -\frac{F_\tau}{F_D} > 0.$$

Thus the lower-opportunity-cost seller's sale probability is strictly increasing in τ, while the higher-opportunity-cost seller's sale probability is strictly decreasing.

The higher opportunity cost seller's markup, $\Phi(1-D,\tau)$, falls because $1-D$ falls and Φ is increasing in its first argument and decreasing in τ. By rearranging equation (36), the lower opportunity cost seller's markup can be written as

$$\Phi(D,\tau) = \widehat{c}_j - \widehat{c}_i + \Phi(1-D,\tau) + \sigma \log \frac{1-D}{D}.$$

The right-hand side falls with τ, because $\Phi(1-D,\tau)$ falls and D rises. Prices move one-for-one with markups because opportunity costs are fixed.

Part 3. Suppose again that $\widehat{c}_i < \widehat{c}_j$. The argument above showed that D_i^* is strictly increasing in τ. Therefore

$$\Phi(D,\tau) - \Phi(1-D,\tau) = \widehat{c}_j - \widehat{c}_i + \sigma \log \frac{1-D}{D}$$

falls with τ. Hence the lower-opportunity-cost seller's markup falls by more than the higher-opportunity-cost seller's markup. ■

B.4 Proof of Proposition 3

Proposition 3 *Consider the model where $m = M = 2$. In state $\mathbf{e} = (1, 2)$,*

1. *the derivative of $DI_2(\mathbf{e}, \tau) - DI_1(\mathbf{e}, \tau)$ with respect to τ, evaluated at $\tau = 0$, is greater than 0.*
2. *$D_2^*(\mathbf{e}) > \frac{1}{2}$ for all τ, and $D_2^*(\mathbf{e})$ monotonically increases in τ.*
3. *the probability that the industry is in state (1,2) from the third period onwards is increasing in τ.*

Proof. Continue to use the $M = 2$ notation from the proof of part 3 of Proposition 1:

$$\Delta_1 := \Delta(1), \qquad \Delta_2 := \Delta(2), \qquad \Delta c := c(1) - c(2) > 0.$$

Let

$$x(\tau) := D_1^*(1, 2; \tau)$$

be the laggard's sale probability in the asymmetric state, and set

$$\Pi(D, \tau) := D\Phi(D, \tau).$$

The proof of part 3 of Proposition 1 shows that $x(\tau) \in (0, 1/2)$ is the unique solution to

$$\Delta c = \Psi(x, \tau).$$

The same proof also establishes

$$\Psi_x(x, \tau) < 0 \qquad \text{for all } x \in (0, 1/2).$$

We use these facts below.

<u>*Part 1.*</u> Let

$$R(\tau) := DI_2(1,2;\tau) - DI_1(1,2;\tau).$$

Using the logit inversion and the pricing rule in state $(1,2)$, the equilibrium condition can be rewritten as

$$R(\tau) = \sigma \log \frac{1-x(\tau)}{x(\tau)} - \Phi(x(\tau),\tau) + \Phi(1-x(\tau),\tau) - \Delta c. \tag{37}$$

Set

$$y(\tau) := \frac{1-x(\tau)}{x(\tau)} > 1.$$

For a generic $y > 1$, define

$$S(y,\tau) := \frac{1}{\sigma}\left[\sigma \log y - \Phi\left(\frac{1}{1+y},\tau\right) + \Phi\left(\frac{y}{1+y},\tau\right)\right],$$

and

$$K(y,\tau) := \frac{1}{\sigma}\left[\eta_{1/2}\Pi(1/2,\tau) + \eta_x\Pi\left(\frac{1}{1+y},\tau\right) + \eta_{1-x}\Pi\left(\frac{y}{1+y},\tau\right)\right].$$

Then the equilibrium equation is

$$\frac{\Delta c}{\sigma} = S(y,\tau) + \beta K(y,\tau),$$

which implicitly determines $y(\tau)$. By (37),

$$\frac{R(\tau)}{\sigma} = S(y(\tau),\tau) - \frac{\Delta c}{\sigma}.$$

Differentiating the equilibrium equation gives

$$y'(\tau) = -\left.\frac{S_\tau + \beta K_\tau}{S_y + \beta K_y}\right|_{(y(\tau),\tau)}.$$

Differentiating the expression for $R(\tau)$ and substituting this formula at $\tau = 0$ gives

$$\frac{R'(0)}{\sigma} = \beta \left.\frac{S_\tau K_y - S_y K_\tau}{S_y + \beta K_y}\right|_{(y_0,0)}, \qquad y_0 := y(0). \tag{38}$$

The denominator is positive because

$$S_y + \beta K_y = \frac{\partial}{\partial y}\left[\frac{\Psi\left(\frac{1}{1+y}, \tau\right)}{\sigma}\right],$$

and the proof of Proposition 1 gives $\Psi_x < 0$, while $\partial[1/(1+y)]/\partial y < 0$.

It remains to sign the numerator. For $y > 1$, define

$$\ell_1(y) := \log\left(\frac{1+y}{y}\right), \qquad \ell_2(y) := \log(1+y),$$

and

$$C_1(y) := \frac{1 + 1/y + 1/y^2}{\log 2} - \frac{1}{y^2 \ell_1(y)} - \frac{1+y}{y \ell_2(y)},$$

$$F(y) := \frac{2(1 + 1/y + 1/y^2)}{\log 2} - \frac{y+2}{y^2 \ell_1(y)} - \frac{2y+1}{y \ell_2(y)}.$$

At $\tau = 0$, substituting $\Phi(D, 0) = \sigma/(1-D)$, using equations (29), (30), (31), and $\eta_{1/2} = -(\eta_x + \eta_{1-x})$, gives

$$(S_\tau K_y - S_y K_\tau)|_{(y,0)} = \frac{\beta(\Delta_1 + \Delta_2 - 1)^2}{(1 - \beta\Delta_1)[1 - \beta(1 - \Delta_2)]} C_1(y) - \eta_{1-x} F(y). \tag{39}$$

Recall that $\eta_{1-x} \leq 0$ by (30), so it suffices to verify that $C_1(y) > 0$ and $F(y) > 0$. Set

$$a := \frac{1}{y} \in (0,1), \qquad L := \log 2, \qquad L_1 := \log(1+a), \qquad L_2 := \log(1 + 1/a).$$

Since $L_1 < L < L_2$, define the positive quantities

$$U := \frac{1}{L_1} - \frac{1}{L}, \qquad V := \frac{1}{L} - \frac{1}{L_2}.$$

It can be verified that the following single-variable function of $a \in (0,1)$ is positive

$$\frac{1+a}{L} - \frac{a}{L_1} - \frac{1}{L_2} > 0, \tag{40}$$

which is equivalent to $V > aU$. Given $V > aU$, rewriting C_1 and F in terms of a gives

$$C_1 = \frac{1+a+a^2}{L} - \frac{a^2}{L_1} - \frac{1+a}{L_2} = (1+a)V - a^2U > aU > 0,$$

and

$$F = \frac{2(1+a+a^2)}{L} - \frac{a+2a^2}{L_1} - \frac{2+a}{L_2} = (2+a)V - a(1+2a)U > a(1-a)U > 0.$$

<u>*Part 2.*</u> We next prove that $x(\tau)$ is strictly decreasing in τ. The argument is for $\tau < 1$; the endpoint $\tau = 1$ follows by continuity. By the implicit function theorem,

$$x'(\tau) = -\frac{\Psi_\tau(x(\tau),\tau)}{\Psi_x(x(\tau),\tau)}.$$

Since $\Psi_x < 0$, it is enough to prove $\Psi_\tau < 0$.

Using (29) and

$$\partial_\tau \Pi(D,\tau) = -DM(D,\tau),$$

differentiating $\Psi(x,\tau)$ at fixed x gives

$$\Psi_\tau = A\,M(x,\tau) - B\,M(1/2,\tau) - C\,M(1-x,\tau), \tag{41}$$

where

$$A := 1 - \beta\eta_x x, \qquad B := \frac{\beta\eta_{1/2}}{2}, \qquad C := 1 + \beta\eta_{1-x}(1-x).$$

The coefficient signs in statements (31) and (30) imply

$$A > 0, \qquad C \geq 1 - \beta(1-x) > 0.$$

A direct simplification using the coefficients from the proof of Proposition 1 gives

$$B + C - A = \frac{\beta^2(1-2x)(\Delta_1 + \Delta_2 - 1)^2}{2(1-\beta\Delta_1)[1-\beta(1-\Delta_2)]} \geq 0. \tag{42}$$

Since $x < 1/2$, Lemma B.1 implies

$$M(x,\tau) < M(1/2,\tau) < M(1-x,\tau).$$

Using this ordering in (41),

$$\Psi_\tau = A\{M(x,\tau) - M(1/2,\tau)\} - (B+C-A)M(1/2,\tau) - C\{M(1-x,\tau) - M(1/2,\tau)\} < 0.$$

Therefore $x'(\tau) < 0$. Since $D_2^*(1,2;\tau) = 1 - x(\tau)$, we have $D_2^*(1,2;\tau) > 1/2$, and $D_2^*(1,2;\tau)$ is strictly increasing in τ.

Part 3. It remains to translate Part 2 into a statement about state probabilities. Fix $x \in (0,1/2)$, and aggregate the four states into

$$\mathcal{U} := (1,1), \qquad \mathcal{A} := \{(1,2),(2,1)\}, \qquad \mathcal{V} := (2,2).$$

Given the laggard sale probability x, the induced transition matrix over these aggregate states, ordered as $(\mathcal{U}, \mathcal{A}, \mathcal{V})$, is

$$P_x = \begin{pmatrix} \Delta_1 & 1-\Delta_1 & 0 \\ x\Delta_1\Delta_2 & 1-x\alpha & x(1-\Delta_1)(1-\Delta_2) \\ 0 & \Delta_2 & 1-\Delta_2 \end{pmatrix}, \qquad \alpha := \Delta_1\Delta_2 + (1-\Delta_1)(1-\Delta_2) > 0. \quad (43)$$

Let $e_{\mathcal{A}} = (0,1,0)'$, and define

$$f_n(x) := P_x^n e_{\mathcal{A}}.$$

The three elements of $f_n(x)$ are the n-step probabilities of being in the aggregate asymmetric state $\mathcal{A}$ when the current aggregate state is, respectively, $\mathcal{U}$, $\mathcal{A}$, or $\mathcal{V}$. Let $a_t(x)$ denote the probability of being in $\mathcal{A}$ in period t, starting from $\mathcal{U}$ in period 1. Then

$$a_t(x) = f_{t-1}^{\mathcal{U}}(x).$$

Differentiate $f_{n+1} = P_x f_n$:

$$f'_{n+1} = \frac{\partial P_x}{\partial x} f_n + P_x f'_n, \qquad f'_0 = 0.$$

A direct calculation gives

$$\frac{\partial P_x}{\partial x} f_n = (0, -r_n, 0)',$$

where

$$r_n := \alpha f_n^{\mathcal{A}} - \Delta_1\Delta_2 f_n^{\mathcal{U}} - (1-\Delta_1)(1-\Delta_2) f_n^{\mathcal{V}}.$$

Set

$$h_n := f_n^{\mathcal{V}} - f_n^{\mathcal{U}}, \qquad \gamma := \Delta_1 + \Delta_2 - 1.$$

A direct calculation from $f_{n+1} = P_x f_n$ gives

$$h_{n+1} = \theta h_n + \frac{\gamma}{\alpha} r_n, \qquad \theta := \frac{\Delta_1(1-\Delta_2)(1-\Delta_1+\Delta_2)}{\alpha} \geq 0,$$

and

$$r_{n+1} = \phi(x) r_n + \mu\gamma h_n, \qquad \mu := \frac{\Delta_1\Delta_2(1-\Delta_1)(1-\Delta_2)}{\alpha} \geq 0,$$

where

$$\phi(x) := 1 - \alpha x - \frac{(1-\Delta_1)\Delta_2(1+\Delta_1-\Delta_2)}{\alpha}.$$

The function ϕ is decreasing in x, and

$$2\alpha\phi(1/2) = (\Delta_1+\Delta_2-1)^2 + 2\Delta_1(1-\Delta_2)\alpha \geq 0.$$

Hence $\phi(x) \geq 0$ for all $x \in (0, 1/2)$.

The initial conditions are

$$r_0 = \alpha > 0, \qquad \gamma h_0 = 0.$$

We claim that

$$r_n \geq 0, \qquad \gamma h_n \geq 0$$

for every n. The claim holds at $n = 0$. If it holds at n, then

$$\gamma h_{n+1} = \theta\gamma h_n + \frac{\gamma^2}{\alpha} r_n \geq 0,$$

and

$$r_{n+1} = \phi(x) r_n + \mu\gamma h_n \geq 0.$$

The claim follows by induction.

Therefore

$$\frac{\partial P_x}{\partial x} f_n = (0, -r_n, 0)' \le 0$$

componentwise. Since P_x is nonnegative, another induction using

$$f'_{n+1} = \frac{\partial P_x}{\partial x} f_n + P_x f'_n, \qquad f'_0 = 0,$$

gives

$$f'_n(x) \le 0 \qquad \text{componentwise for every } n.$$

The inequality is strict for the $\mathcal{U}$-component from the second transition onward. Indeed,

$$f'_1(x) = (0, -\alpha, 0)',$$

so $(f_1^{\mathcal{A}})' < 0$. If $(f_n^{\mathcal{A}})' < 0$ and $f'_n \le 0$, then

$$(f_{n+1}^{\mathcal{A}})' = -r_n + x\Delta_1\Delta_2 (f_n^{\mathcal{U}})' + (1 - x\alpha)(f_n^{\mathcal{A}})' + x(1-\Delta_1)(1-\Delta_2)(f_n^{\mathcal{V}})' < 0,$$

because $1 - x\alpha > 0$. Hence $(f_n^{\mathcal{A}})' < 0$ for every $n \ge 1$. Since $\Delta_1 < 1$,

$$(f_{n+1}^{\mathcal{U}})' = \Delta_1 (f_n^{\mathcal{U}})' + (1-\Delta_1)(f_n^{\mathcal{A}})' < 0 \qquad \text{for every } n \ge 1.$$

Thus

$$a'_t(x) = (f_{t-1}^{\mathcal{U}})'(x) < 0 \qquad \text{for every } t \ge 3.$$

By symmetry, the two asymmetric states, (1,2) and (2,1), are equally likely when the process starts from $(1,1)$, so the probability of state $(1,2)$ is $a_t(x)/2$. Part 2 shows that $x(\tau)$ is strictly decreasing in τ. Therefore, for every $t \ge 3$, the probability of state $(1,2)$ is strictly increasing in τ. ∎

B.5 Proof of Proposition 4

Proposition 4 *In the $M = 2$ model, TS^{PDV} is strictly monotonically increasing in τ if and only if $\frac{c(1)-c(2)}{\sigma} \geq \Lambda(\beta, \Delta(1), \Delta(2))$ where $\Lambda(\beta, \Delta(1), \Delta(2))$ is a function that decreases in the last two arguments. Otherwise TS^{PDV} is monotonically increasing in τ up to some value $0 < \tau' < 1$, where the equilibrium implements the social optimum, and then it is monotonically decreasing.*

Proof. We first make the cutoff explicit. Define

$$\lambda(\beta, \Delta_1, \Delta_2) := \frac{\Delta_1\Delta_2[1-\beta(1-\Delta_2)]}{\Delta_1\Delta_2[1-\beta(1-\Delta_2)] + (1-\Delta_1)(1-\Delta_2)(1-\beta\Delta_1)},$$

and

$$h(z) := \frac{\log(2/(1+e^{-z}))}{z}, \qquad z > 0.$$

Then

$$\Lambda(\beta, \Delta_1, \Delta_2) := \begin{cases} +\infty, & \text{if } \Delta_1 = 0 \text{ or } \Delta_2 = 0, \\ h^{-1}(\lambda(\beta, \Delta_1, \Delta_2)), & \text{if } 0 < \lambda(\beta, \Delta_1, \Delta_2) < \frac{1}{2}, \\ 0, & \text{if } \lambda(\beta, \Delta_1, \Delta_2) \geq \frac{1}{2}. \end{cases}$$

The function h is strictly decreasing from $1/2$ to 0. The odds ratio for λ is

$$\frac{\lambda}{1-\lambda} = \frac{\Delta_1\Delta_2[1-\beta(1-\Delta_2)]}{(1-\Delta_1)(1-\Delta_2)(1-\beta\Delta_1)},$$

which is increasing in Δ_1 and Δ_2. Hence Λ decreases in Δ_1 and Δ_2.

We next study total surplus as a function of a fixed laggard sale probability x in asymmetric states. Aggregate the states into $\mathcal{U} = (1,1)$, $\mathcal{A} = \{(1,2),(2,1)\}$, and $\mathcal{V} = (2,2)$. Recall that the induced transition matrix P_x is given by (43). Normalize $c(2) = 0$; this only subtracts a constant from total surplus. The one-period total-surplus vector is

$$s(x) = \begin{pmatrix} \sigma \log 2 - \Delta c \\ \sigma \left[x \log \frac{1}{x} + (1-x) \log \frac{1}{1-x} \right] - x\Delta c \\ \sigma \log 2 \end{pmatrix}.$$

The Bellman equation for total surplus is $W = s(x) + \beta P_x W$. The $\mathcal{U}$- and $\mathcal{V}$-equations give

$$W_{\mathcal{U}} = \frac{s_{\mathcal{U}}}{1-\beta\Delta_1} + U_1 W_{\mathcal{A}}, \text{ where } U_1 := \frac{\beta(1-\Delta_1)}{1-\beta\Delta_1}$$
$$W_{\mathcal{V}} = \frac{s_{\mathcal{V}}}{1-\beta(1-\Delta_2)} + \frac{\beta\Delta_2}{1-\beta(1-\Delta_2)} W_{\mathcal{A}}.$$

Substituting these into the $\mathcal{A}$-equation gives

$$W_{\mathcal{A}}(x) = \frac{s_{\mathcal{A}}(x) + \beta x R}{1-\beta+\beta x M},$$

where

$$\omega_{\mathcal{U}} := \frac{\Delta_1\Delta_2}{1-\beta\Delta_1}, \qquad \omega_{\mathcal{V}} := \frac{(1-\Delta_1)(1-\Delta_2)}{1-\beta(1-\Delta_2)}, \qquad M := (1-\beta)(\omega_{\mathcal{U}} + \omega_{\mathcal{V}}),$$

$$R := \omega_{\mathcal{U}}(\sigma \log 2 - \Delta c) + \omega_{\mathcal{V}} \sigma \log 2.$$

Therefore

$$TS^{PDV}(x) = W_{\mathcal{U}}(x) = \frac{\sigma \log 2 - \Delta c}{1-\beta\Delta_1} + U_1 \frac{\sigma \left[x \log \frac{1}{x} + (1-x) \log \frac{1}{1-x} \right] - x\Delta c + \beta x R}{1-\beta+\beta x M}.$$

Let $K := 1 - \beta + \beta M$. Differentiating $W_{\mathcal{U}}$ gives

$$\frac{dTS^{PDV}(x)}{dx} = \frac{U_1}{(1-\beta+\beta x M)^2} \Xi(x), \tag{44}$$

where $U_1 > 0$, $M > 0$, $K > 0$, and

$$\Xi(x) = (1-\beta)\sigma \log \frac{1}{x} - K\sigma \log \frac{1}{1-x} - (1-\beta)(\Delta c - \beta R).$$

Hence

$$\Xi'(x) = -\sigma \frac{1-\beta}{x} - \sigma \frac{K}{1-x} < 0.$$

Moreover, $\Xi(x) \to +\infty$ as $x \downarrow 0$, while

$$\Xi\left(\frac{1}{2}\right) = -\Delta c \, \frac{(1-\beta)[1-\beta\Delta_1(1-\Delta_2)]}{1-\beta\Delta_1} < 0.$$

Thus there is a unique $x^{SP} \in (0, 1/2)$ such that $TS^{PDV}(x)$ is strictly increasing on $(0, x^{SP})$ and strictly decreasing on $(x^{SP}, 1)$.

The equilibrium path starts to the right of the planner peak. That is, $x(0) > x^{SP}$. To see this, set $y = (1-x)/x$. At $\tau = 0$, the equilibrium value y_E solves $\Delta c/\sigma = E(y_E)$, where

$$E(y) = \log y + y - \frac{1}{y} + \beta\left(\eta_{1/2} + \frac{\eta_x}{y} + \eta_{1-x} y\right).$$

The planner value y^{SP} solves $\Delta c/\sigma = P(y^{SP})$, where

$$P(y) = \frac{\log y + \beta(\omega_{\mathcal{U}} + \omega_{\mathcal{V}}) \log\left(\frac{2y}{1+y}\right)}{1 + \beta\omega_{\mathcal{U}}}.$$

Both functions equal zero at $y = 1$, and direct differentiation gives $E'(y) - P'(y) > 0$ for every $y > 1$. Therefore $E(y) > P(y)$ for $y > 1$, so the common level $\Delta c/\sigma$ is reached at $y_E < y^{SP}$. Equivalently, $x(0) > x^{SP}$.

By the implicit-function argument in the proof of Proposition 3, $x(\tau)$ is continuously differentiable and strictly decreasing for $\tau < 1$, and it extends continuously to $\tau = 1$. Because $TS^{PDV}(x)$ is single-peaked and the path begins to the right of the peak, $TS^{PDV}(x(\tau))$ is either strictly increasing or strictly increasing and then strictly decreasing. The latter case occurs exactly when the path crosses the peak, i.e., when $x(1) < x^{SP}$.

At $\tau = 1$, markups are zero and prices equal current costs, so

$$x(1) = \frac{1}{1 + \exp(\Delta c/\sigma)}.$$

Since Ξ is strictly decreasing, $x^{SP} > x(1)$ if and only if $\Xi(x(1)) > 0$. Substitution into the expression for Ξ gives

$$\Xi(x(1)) > 0 \quad \Longleftrightarrow \quad \frac{\log(2/(1 + e^{-\Delta c/\sigma}))}{\Delta c/\sigma} > \lambda(\beta, \Delta_1, \Delta_2).$$

Using the definition of h and Λ, this is equivalent to

$$\frac{\Delta c}{\sigma} < \Lambda(\beta, \Delta_1, \Delta_2).$$

Thus TS^{PDV} is strictly hump-shaped in τ exactly under this condition. If the reverse weak inequality holds, the path never crosses the planner peak before $\tau = 1$, and TS^{PDV} is strictly increasing in τ. ∎

C Additional Computational Results

C.1 Illustrations of Changes in Static Markups, Dynamic Incentives, Lead Length and Market Structure for $M = 2$

To illustrate what can happen to static markups, dynamic incentives and market structure when $m = M = 2$, we assume $\beta = \frac{1}{1.05}$, $\kappa = 10$ and $\sigma = 1$, and consider two different technologies. The first technology has $\rho = 0.15$ and $\delta = 0.1$, so the leader has a large cost advantage ($c(1) = 10$ and $c(2) = 1.5$). The second technology has $\rho = 0.6$ and $\delta = 0.05$, so the cost advantage is smaller and there is also a lower probability that the laggard and the leader will switch positions if the laggard makes a sale.

Panels (a) and (d) of Figure C.1 show equilibrium dynamic incentives in the asymmetric state, and the difference between them, i.e., the red line is what we called the DDI in the text. If $\rho = 0.15$ and $\delta = 0.1$, the leader has the stronger dynamic incentives for all $\tau < 1$. On the other hand, if $\rho = 0.6$ and $\delta = 0.05$, it is the laggard that has the stronger dynamic incentives. In symmetric states, the DDI is always equal to zero (not shown). Panels (b) and (e) show equilibrium static markups in the asymmetric state, as functions of τ, and, for comparison, the static markups in the symmetric states (1,1) and (2,2).

In symmetric states, static markups decline monotonically in τ as bargaining power does not affect the DDI. In (1,2) the markups would decline monotonically if the DDI were fixed, but the leader's static markup can increase, as it does in panel (b) for small τ, when the change in the DDI increases the probability that the leader sells.[51]

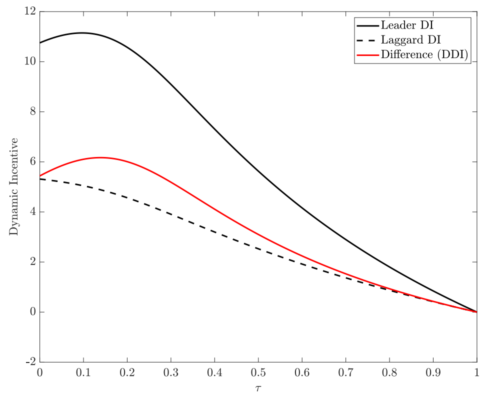
For both technologies, the laggard's dynamic incentive falls monotonically as τ increases, while the leader's dynamic incentive initially increases as seller 2's lead is expected to last longer. Consistent with Proposition 3, the DDI increases with τ for both of these technologies when τ is small. Even though the DDI in panel (a) declines for higher values of τ, the static markup effect is large enough that the probability that the leader wins in the asymmetric state monotonically increases in τ. Panels (c) and (f) show that, as τ increases, the number of periods that the leader expects its lead to last in equilibrium increases rapidly, for both technologies, even though the expected lead lengths when $\tau = 0$ are somewhat different (around 12.5 periods in (c) and less than 3 periods in (f)).

[51] Further numerical analysis identifies examples where the leader's dynamic incentive decreases in τ when $\tau = 0$.

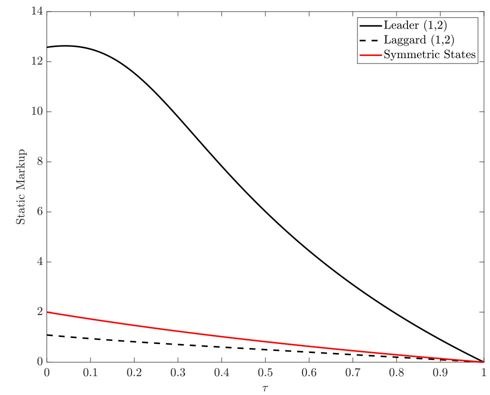
Figure C.1: Equilibrium Static Markups, Dynamic Incentives, Sale Probabilities and Expected Lead Length in the $M = 2$ Model for $(\rho = 0.15, \delta = 0.1)$ and $(\rho = 0.6, \delta = 0.05)$ as Functions of τ. These figures assume that, as for our $M = 30$ specification, $\beta = \frac{1}{1.05}$, $\kappa = 10$ and $\sigma = 1$.

$\underline{\rho = \mathbf{0.15}, \delta = \mathbf{0.1}}$

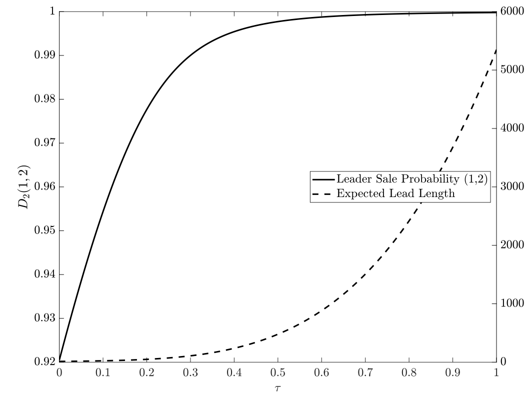
(a) Dynamic Incentives in State (1,2)



(b) Static Markups

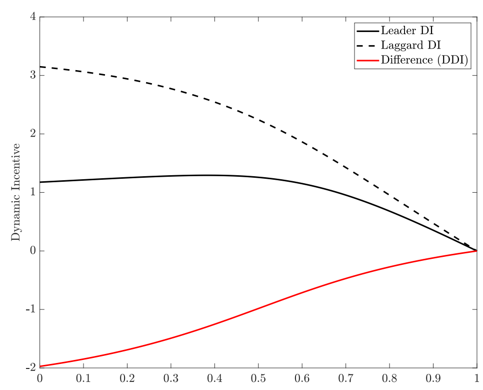


(c) Leader Sale Probability (1,2) and Expected Lead Length

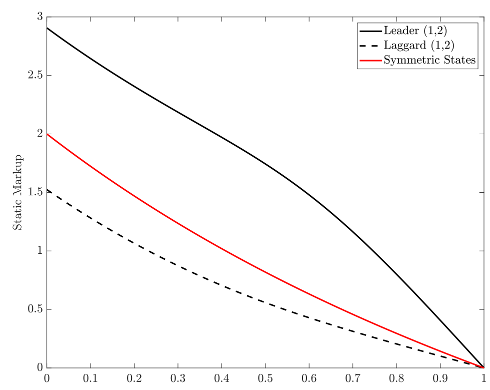


$\underline{\rho = \mathbf{0.6}, \delta = \mathbf{0.05}}$

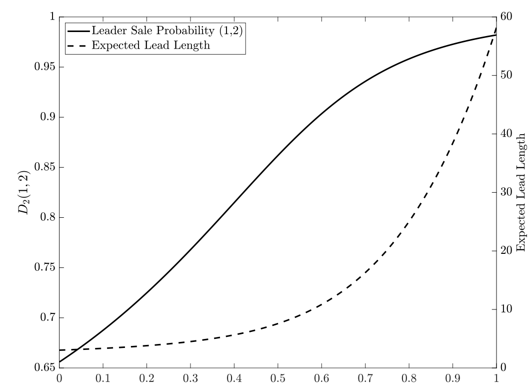
(d) Dynamic Incentives in State (1,2)



(e) Static Markups



(f) Leader Sale Probability (1,2) and Expected Lead Length



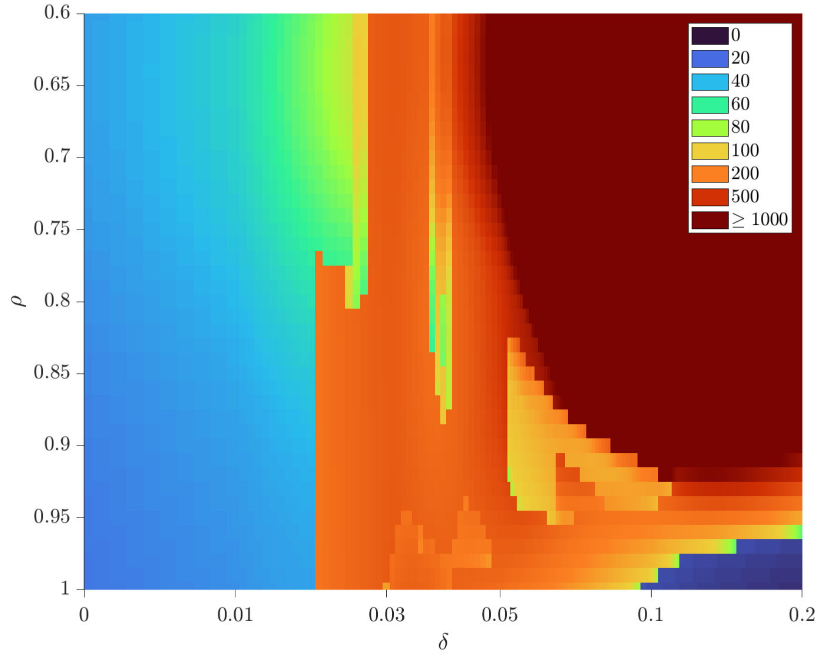
C.2 $M = 30$ Specification Lead Lengths

Figure C.2 shows the expected number of periods that a leader in period 32 will expect its lead to last if $\tau = 0$. This figure complements the text Figure 3(a) which shows expected concentration in the same period, but the expected lead length provides additional information about how market structure is expected to evolve in later periods. The figure reinforces how there is a difference between technologies where depreciation is fairly limited, allowing a laggard to catch up quite quickly, and technologies where know-how depreciation is more likely, which raises the leader's expected lead length to more than 200 periods.

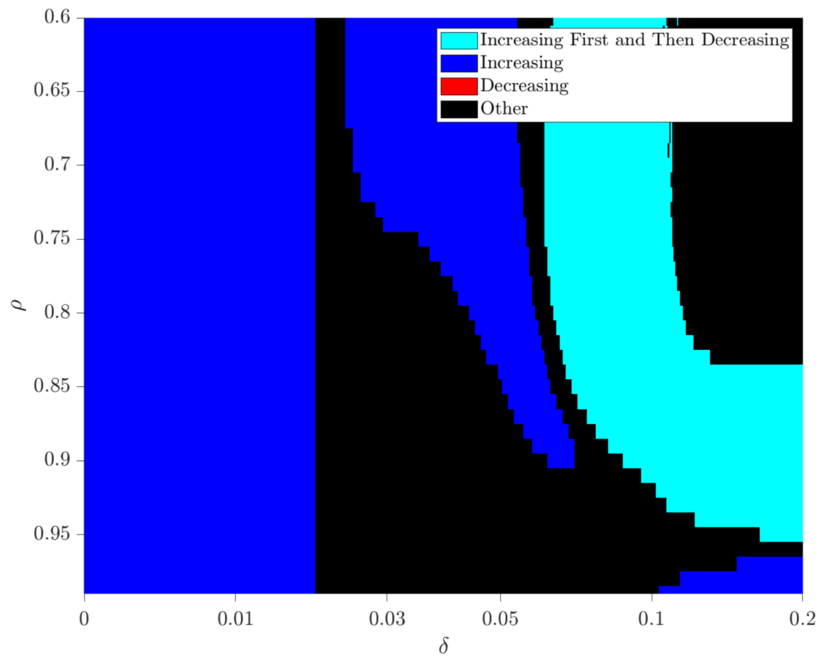
We can also analyze whether lead lengths are increasing in τ. We know that in the $M = 2$ model the number of periods that a leader will expect its lead to last will increase monotonically in τ as τ rises. Figure C.2(b) shows what happens, given equilibrium play, to the number of periods that a leader in period 32 will expect its lead to last. Of course, as τ increases, the distribution of asymmetric states in which the leader will be will change. When depreciation is not too likely, and equilibria are unique, the expected lead length increases monotonically. When depreciation is more likely, the pattern is more complicated, partly because of the existence of multiple equilibria when τ is small. However, when LBD is significant (e.g., $\rho < 0.8$) lead lengths are typically either monotonically increasing, or monotonically increasing and then monotonically decreasing, in τ.

Figure C.2: Buyer Bargaining Power and Expected Lead Length For Leaders in the 32nd Period of the Game in the $M = 30$ Specification

(a) Expected Lead Length if $\tau = 0$. If there are multiple equilibria we report the longest expected lead length.



(b) Monotonicity of the Number of Additional Periods that a Leader in Period 32 Expects Its Lead to Last as τ Increases. If there are multiple equilibria, we use the longest expected lead length for each level of τ.

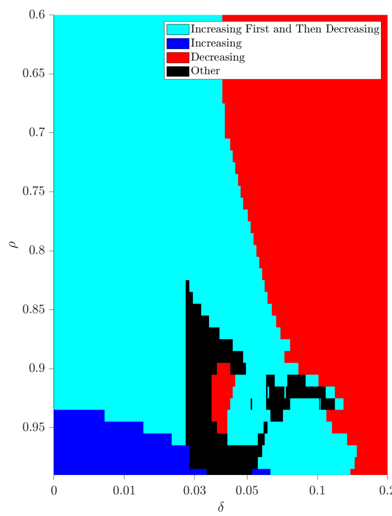


C.3 $M = 30$ Figures Using the Minimum Value of the Statistic when there are Multiple Equilibria

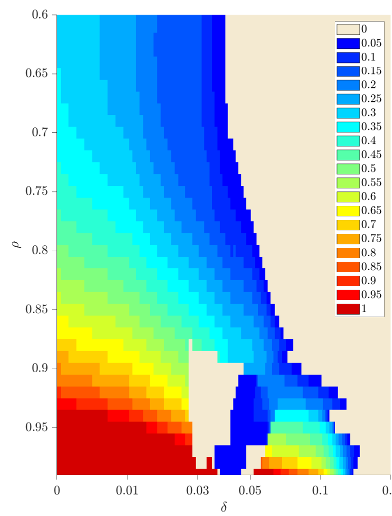
The heatmap figures in text Section 4 use the maximum value of the relevant statistic across equilibria when multiple equilibria exist for small values of τ in the $M = 30$ specification. In this Appendix, Figure C.3 presents figures that instead use the *minimum* value of the statistic, so that the reader can confirm that the results are similar.

Figure C.3: Figures for the $M = 30$ Specification Where the Minimum Value of the Statistic Across Equilibria is Used when Multiple Equilibria Exist

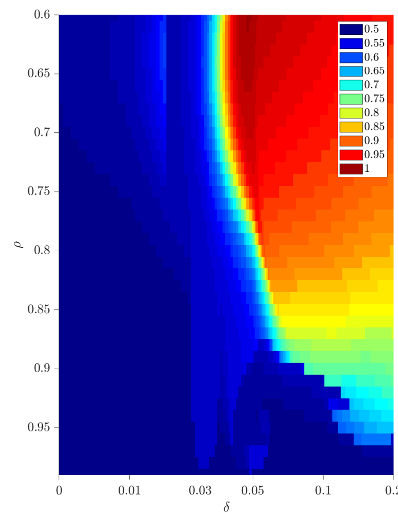
(a) Monotonicity of
Leader DI^{PDV} - Laggard DI^{PDV}
Compare Fig. 2(a)



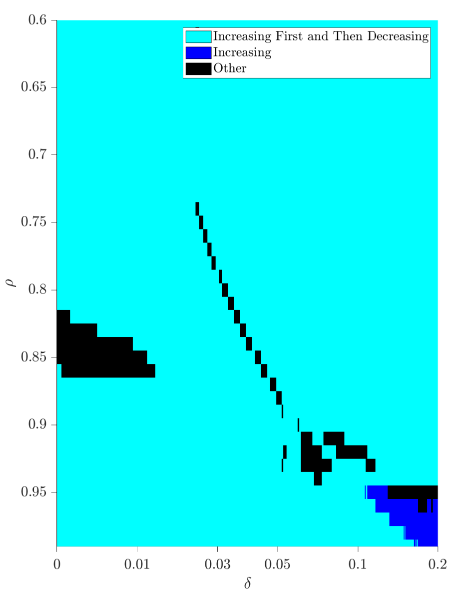
(b) Value of τ Maximizing
Leader DI^{PDV} - Laggard DI^{PDV}
Compare Fig. 2(b)



(c) Value of HHI^{32}
for $\tau = 0$
Compare Fig. 3(a)



(d) Monotonicity of
TS^{PDV}
Compare Fig. 5(a)



(e) Value of τ Maximizing
TS^{PDV}
Compare Fig. 5(b)

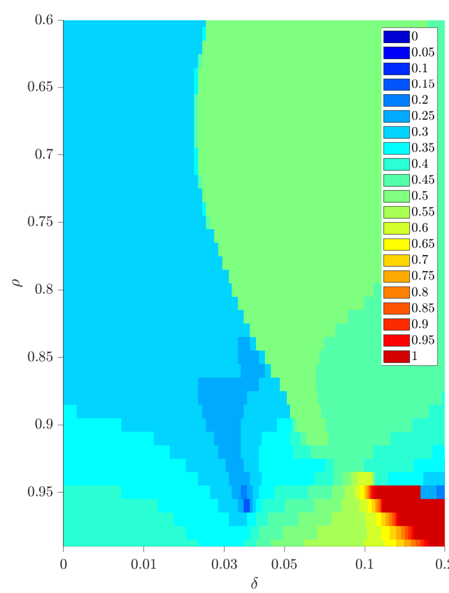
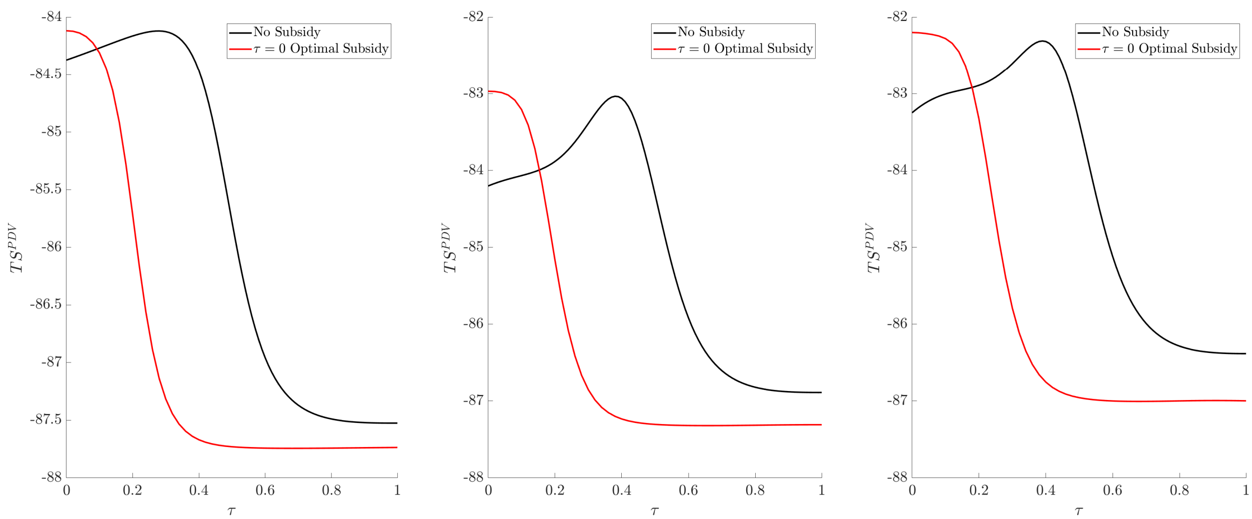


Figure C.4: Efficiency as a Function with No Transfer Scheme and the $\tau = 0$ Optimal Transfer Scheme in a Specification where $M = 7$, $m = 5$, $\rho = 0.65$, $\delta = 0.03$, $\sigma = 1$ and $\kappa = 10$.

(a) $N = 2$ (b) $N = 3$ (c) $N = 4$



C.4 The Effect of $\tau = 0$ Subsidies in an Alternative Parameterization

In Section 5.1, we use one particular parameterization of the $M = 30$ specification to illustrate how, when sellers act dynamically, a transfer scheme that would be optimal if sellers set prices can lower welfare if buyers have even small amounts of bargaining power. A feature of that example is that the $\tau = 0$ transfer scheme supports multiple equilibria even $\tau = 0$.

In this Appendix, we consider the effect of transfer schemes in the parameterization that we use to analyze horizontal mergers. In this set-up, $M = 7$, $m = 5$, $\rho = 0.65$, $\delta = 0.03$, $\sigma = 1$ and $\kappa = 10$. We analyze cases with 2, 3 or 4 single-product firms.

Figure C.4 shows the results. In each panel, the red line shows TS^{PDV} under the transfer scheme that is optimal when $\tau = 0$. There is no multiplicity in this parameterization so that the level of TS^{PDV} when $\tau = 0$ is the level of expected efficiency that the social planner would achieve. The black line shows the level of welfare that is achieved in equilibrium when there is no transfer scheme. It is worth noting that, consistent with our discussion in the text, market equilibria come close to acheieving the social planner level of efficiency for a level of τ that is between 0.4 and 0.5. We see in each panel that the $\tau = 0$ optimal scheme lowers efficiency, relative to no scheme at all, when buyers have moderate bargaining power. The threshold for the scheme lowering efficiency is $\tau \geq 0.1$ if $N = 2$, 0.16 if $N = 3$ and 0.18 if $N = 4$.

C.5 Extension of the Model to Allow for Additional and Multiproduct Sellers

Our baseline model assumes single product duopolists. However, our horizontal merger analysis allows for more sellers and for a merger that creates a multiproduct seller. In the Appendix we detail the equations that we can use to solve this more general model.

Consider an industry where K products are sold by N firms. The products are indexed by the set $\mathcal{K} = \{1, 2, \ldots, K\}$, and the firms are indexed by the set $\mathcal{N} = \{1, 2, \ldots, N\}$. Selling firm $n \in \mathcal{N}$ owns a set A_n of products, and $\bigcup_{n \in \mathcal{N}} A_n = \mathcal{K}$.

Bargaining First-Order Condition with Multiproduct Firms. The buyer bargains over a set of prices, $\mathbf{p}_{A_n} = \{p_k : k \in A_n\}$, with seller $n \in \mathcal{N}$. Negotiated prices will be the solutions to the following problem

$$\max_{\mathbf{p}_{A_n}} \left[\sigma \log \sum_k \exp\left(\frac{v - \alpha p_k}{\sigma}\right) - \sigma \log \sum_{k \notin A_n} \exp\left(\frac{v - \alpha p_k}{\sigma}\right)\right]^{\tau} \times \left[\sum_{k \in A_n} D_k(\mathbf{p})(p_k - c_k + \mu_k^n) + \sum_{k \notin A_n} D_k(\mathbf{p})\mu_k^n - \sum_{k \notin A_n} D_k^{-n}(\mathbf{p}_{-n})\mu_k^n\right]^{1-\tau}, \tag{45}$$

where μ_k^n is the continuation value of seller n when the buyer purchases product k.

$$D_k(\mathbf{p}) = \frac{\exp\left(\frac{v-\alpha p_k}{\sigma}\right)}{\sum_j \exp\left(\frac{v-\alpha p_j}{\sigma}\right)}, D_k^{-n}(\mathbf{p}_{-n}) = \frac{\exp\left(\frac{v-\alpha p_k}{\sigma}\right)}{\sum_{j \notin A_n} \exp\left(\frac{v-\alpha p_j}{\sigma}\right)}.$$

Note that for $k \notin A_n$, $D_k(\mathbf{p}) = (1 - D_{A_n}(\mathbf{p}))D_k^{-n}(\mathbf{p}_{-n})$, where $D_{A_n}(\mathbf{p})$ denotes the total market share of firm n:

$$D_{A_n}(\mathbf{p}) = \sum_{k \in A_n} D_k(\mathbf{p}).$$

The bargaining problem can be rewritten as

$$\max_{\mathbf{p}_{A_n}} \left[-\sigma \log\left(1 - D_{A_n}(\mathbf{p})\right)\right]^{\tau} \left[\sum_{k \in A_n} D_k(\mathbf{p})(p_k - \widehat{c}_k)\right]^{1-\tau}, \tag{46}$$

where $\widehat{c}_k$ denotes the "effective" or continuation-adjusted marginal cost. That is,

$$\widehat{c}_k = c_k - \mu_k^n + \sum_{j \notin A_n} D_j^{-n}(\mathbf{p}_{-n})\mu_j^n. \tag{47}$$

The FOC for p_j, with $j \in A_n$, is

$$-\tau \sum_{k \in A_n} D_k(\mathbf{p})(p_k - \widehat{c_k}) + (1-\tau)\left[\frac{\sigma}{\alpha} - (p_j - \widehat{c_j}) + \sum_{k \in A_n} D_k(\mathbf{p})(p_k - \widehat{c_k})\right] \log \frac{1}{1 - D_{A_n}} = 0. \quad (48)$$

Corollary C.1 *The markup over effective marginal cost $p_k - \widehat{c_k}$ is the same across products within one firm. Specifically, for all $k \in A_n$,*

$$p_k - \hat{c}_k = \Phi(D_{A_n}) := \frac{\frac{\sigma}{\alpha}(1-\tau)\log \frac{1}{1-D_{A_n}}}{\tau D_{A_n} + (1-\tau)(1-D_{A_n})\log \frac{1}{1-D_{A_n}}}. \quad (49)$$

This corollary extends to the bargaining model the well-known Bertrand property that, when there is multinomial logit demand, a multiproduct firm will set its price so that there is an identical markup on all of its products.

The equilibrium prices p_k, $k \in \mathcal{K}$, and firm values V_n^S, $n \in \mathcal{N}$, can be solved from (49) together with

$$V_n^S = \sum_{k \in A_n} D_k(\mathbf{p})(p_k - c_k) + \sum_{k \in \mathcal{K}} D_k(\mathbf{p})\mu_k^n,$$

or equivalently,

$$V_n^S = \sum_{k \in A_n} D_k(\mathbf{p})(p_k - \hat{c}_k) + \sum_{k \notin A_n} \mu_k^n. \quad (50)$$

There are $(K+N) \times M^K$ equations in total (K prices and N firm values for each one of the M^K states).

Choice Probability Formulation of Equilibrium Equations. It is also possible to formulate the equilibrium in terms of buyer choice probabilities. Specifically, from (50), we have that

$$\mathbf{V}_n^S = \left(\mathbf{I} - \beta \sum_{k \notin A_n} \mathbf{Q}_k\right)^{-1} \left[\mathbf{D}_{A_n} \circ \mathbf{\Phi}(\mathbf{D}_{A_n})\right].$$

Plugging this into (9) yields that

$$\mathbf{p}_k = \mathbf{\Phi}(\mathbf{D}_{A_n}) + \mathbf{c}_k - \beta \left(\mathbf{Q}_k - \sum_{j \notin A_n} \mathbf{D}_j^{-n} \circ \mathbf{Q}_j \right) \left(\mathbf{I} - \beta \sum_{k \notin A_n} \mathbf{Q}_k \right)^{-1} \left[\mathbf{D}_{A_n} \circ \mathbf{\Phi}(\mathbf{D}_{A_n}) \right].$$

Consequently, prices and firm values can be pinned down from the *aggregate* market shares at firm level, D_{A_n}. So to characterize the equilibrium, it suffices to solve for D_{A_n} from the following equation:

$$D_{A_n}(\mathbf{p}) = \sum_{k \in A_n} \frac{\exp\left(\frac{v - \alpha p_k}{\sigma}\right)}{\sum_j \exp\left(\frac{v - \alpha p_j}{\sigma}\right)}.$$

There are $N \times M^K$ equations in total (N firm shares for each one of the M^K states).

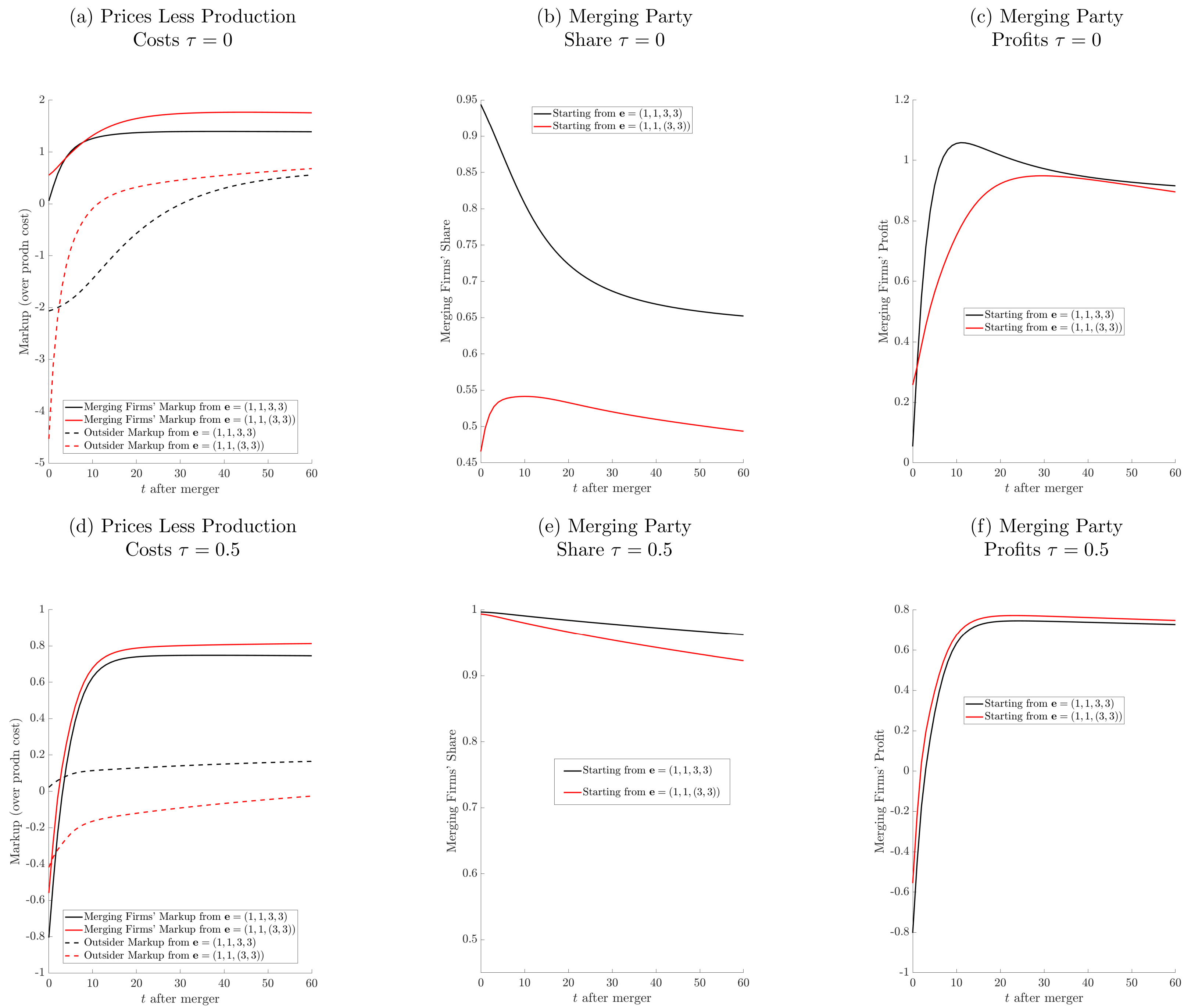
C.6 Further Analysis of Horizontal Mergers

In Section 5.2, we illustrate how, given dynamic equilibrium behavior, the level of buyer bargaining power can affect how a horizontal merger between know-how leaders changes the profits (values) of the merging firms and the welfare impact of the merger. In this Appendix, we look in more detail at the margin and market share dynamics that lead to these effects, and show that dynamics and buyer bargaining power also interact, in interesting ways, in determining the efficiency effects of leader-laggard and laggard-laggard mergers.

Margin and Share Dynamics After a Leader-Leader Merger. Figure C.5 shows how prices and production costs, the merging parties' combined shares and the merging parties' combined profits evolve, on average, over the sixty periods following the merger for $\tau = 0$ (when the merger is not profitable and it decreases efficiency and buyer surplus) and $\tau = 0.5$ (when it is profitable and approximately surplus neutral).

The pre-merger combined market shares of the parties are close to 1 (0.94 and 0.99) in state (1,1,3,3) whether $\tau = 0$ or $\tau = 0.5$. However, absent the merger, there is a difference in how market structure is expected to evolve. If $\tau = 0$, the laggards charge negative markups, and are more likely to sell than when $\tau = 0.5$, so that the leaders' know-how advantage tends to fall over time, and the leaders' charge larger markups and make higher per-period profits while their lead is eroding, as they anticipate they will not have an advantage in the future. The merger not only incentivizes the merged firm to raise prices, but the anticipated softening of competition also incentivizes the non-merging firms to try to catch up even more quickly, by setting very low prices, so that the

Figure C.5: Comparison of Post-Proposed Merger Dynamics for $\tau = 0$ and $\tau = 0.5$ for a Merger Between the Two Leaders in State (1,1,3,3). $M = 7$, $m = 5$, $\rho = 0.65$ and $\delta = 0.03$.



merged firm's share immediately drops to around 50% and stays there. Of course, in a static model, if the merged firm raised prices, its rivals would do so as well.

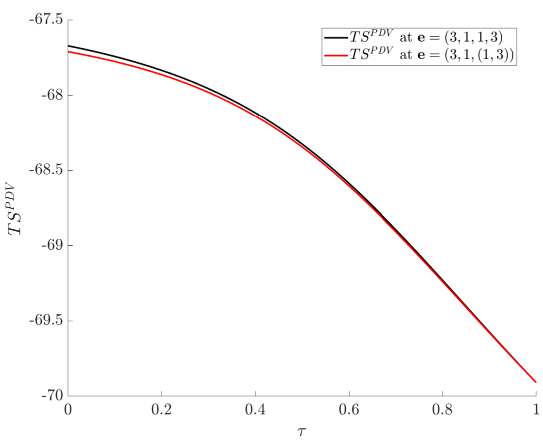
The immediate loss of share makes the merger unprofitable, and, as the changes slow how quickly the leading products will lower their costs, efficiency is also reduced. In contrast, if $\tau = 0.5$, the merged firm's markups are constrained by the buyer's bargaining power, and the merger leads to a smaller increase in the incentive of the laggards to try to catch up, and a much smaller reduction in their prices. Therefore, the merged firm will tend to preserve its lead, in both know-how and share, for a relatively long period of time, and its higher prices, both when it is learning and when its reaches the bottom of its cost curve, will tend to increase the merged firm's profits.

Efficiency Effects After Leader-Laggard and Laggard-Laggard Mergers. We have repeated our analysis of whether mergers are profitable and efficient for the other possible two-firm mergers in the same state.

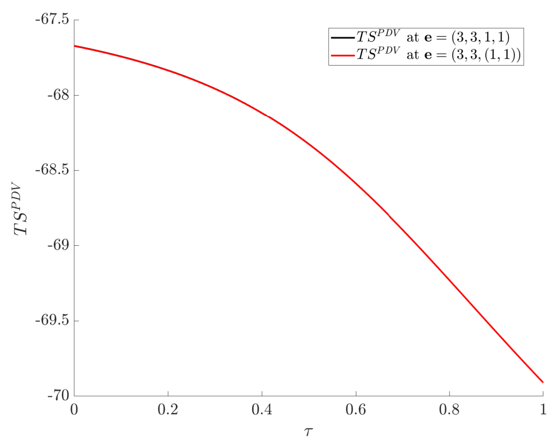
Figure C.6 shows the predicted level of TS^{PDV} after leader-laggard or laggard-laggard mergers, as a function of τ, assuming either static or dynamic seller behavior. The differences are smaller than in the case of the leader-leader merger because, especially when sellers are static, a laggard product is expected to have very few sales (e.g. with static behavior a laggard in state (1,1,3,3) has a 1% probability of making a sale even when $\tau = 0$). When sellers act statically, either merger is (slightly) efficiency reducing for all $\tau < 1$, whereas with dynamic seller behavior, both mergers are efficiency-increasing when τ is small, although, in the case of the leader-laggard merger it is efficiency-reducing for $\tau \geq 0.1$. Therefore, as with a leader-leader merger, the combination of both dynamic seller behavior (i.e. consideration of dynamic incentives) and buyer bargaining power can give different qualitative predictions than when we consider only one of these factors.

Figure C.6: Efficiency After Leader-Laggard and Laggard-Laggard Mergers Assuming Either Static Seller Behavior (panels (a)-(b)) or Dynamic Seller Behavior (panels (c)-(d)) as a Function of τ. $M = 7$, $m = 5$, $\rho = 0.65$ and $\delta = 0.03$. The mergers are proposed in state (1,1,3,3) so that a leader has know-how level 3, and the laggard has know-how level 1. The black lines show outcomes without the merger, and the red lines with the merger.

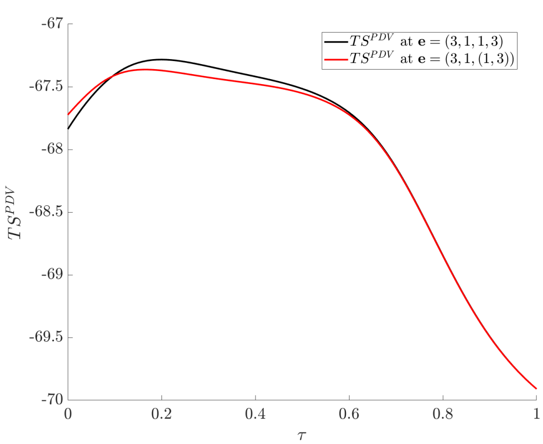
(a) Leader-Laggard Merger
Static: TS^{PDV}



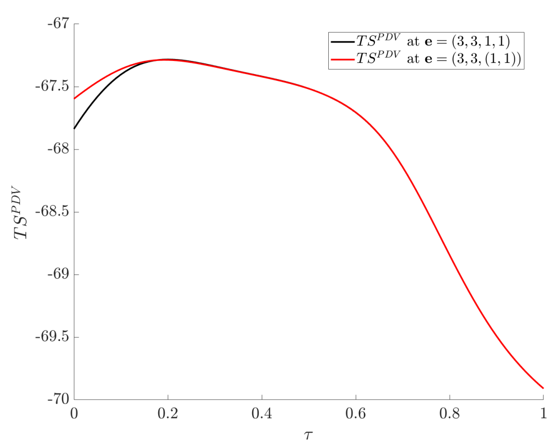
(b) Laggard-Laggard Merger
Static: TS^{PDV}



(c) Leader-Laggard Merger
Dynamic: TS^{PDV}



(d) Laggard-Laggard Merger
Dynamic: TS^{PDV}



References

BESANKO, D., U. DORASZELSKI, Y. KRYUKOV, AND M. SATTERTHWAITE (2010): "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics," *Econometrica*, 78(2), 453–508.